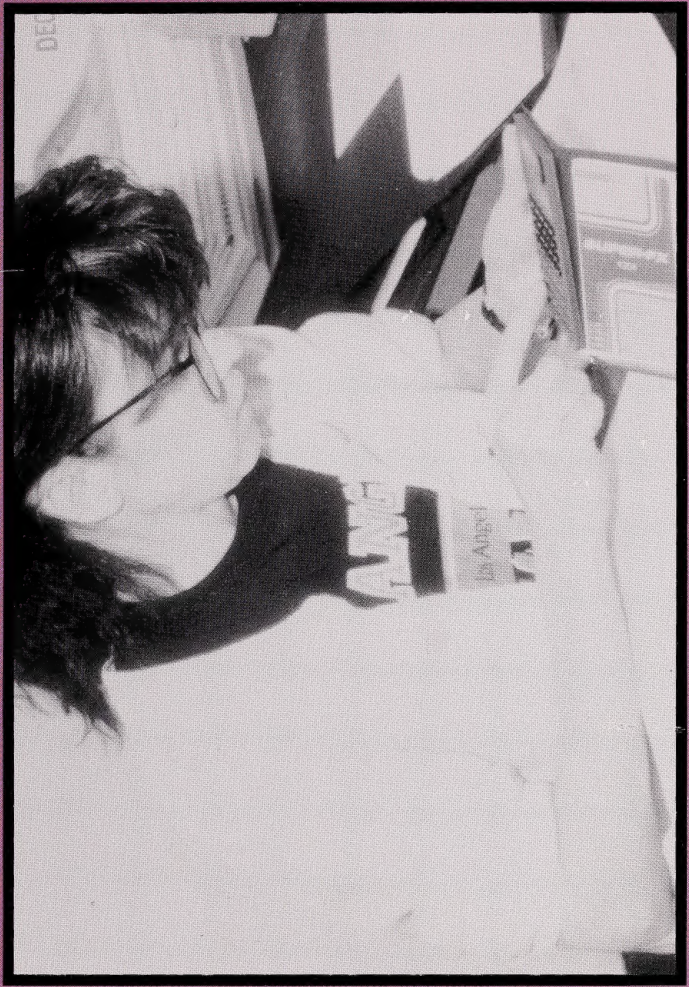


MATHEMATICS

10

UNIT 1



N U M B E R S Y S T E M S



W e l c o m e



Distance Learning

You have chosen an alternate form of learning that allows you to work at your own pace. You will be responsible for your own schedule, for disciplining yourself to study the units thoroughly, and for completing your units regularly. We wish you much success and enjoyment in your studies.

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Mathematics 10 Student Module Unit 1 Number Systems Alberta Distance Learning Centre ISBN No. 0-7741-0741-3

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General Information

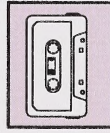
This information explains the basic layout of each booklet.

- **What You Already Know** and **Review** are to help you look back at what you have previously studied. The questions are to jog your memory and to prepare you for the learning that is going to happen in this unit.
- As you begin each **Topic**, spend a little time looking over the components. Doing this will give you a preview of what will be covered in the topic and will set your mind in the direction of learning.
- **Exploring the Topic** includes the objectives, concept development, and activities for each objective. Use your own papers to arrive at the answers in the activities.
- **Extra Help** reviews the topic. If you had any difficulty with **Exploring the Topic**, you may find this part helpful.
- **Extensions** gives you the opportunity to take the topic one step further.
- To summarize what you have learned, and to find instructions on doing the unit assignment, turn to the **Unit Summary** at the end of the unit.
- The **Appendices** include the solutions to **Activities (Appendix A)** and any other charts, tables, etc. which may be referred to in the topics (**Appendix B**, etc.).

Visual Cues

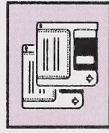
Visual cues are pictures that are used to identify important areas of the material. They are found throughout the booklet.

An explanation of what they mean is written beside each visual cue.



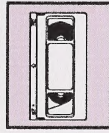
Audiotape

- learning by listening to an audiotape



Computer Software

- learning by using computer software



Videotape

- learning by viewing a videotape



Print Pathway

- choosing a print alternative



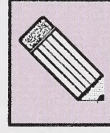
Calculator

- using your calculator



What You Already Know

- reviewing what you already know



Review

- studying previous concepts



Introduction

- introducing the unit



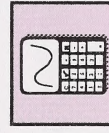
What Lies Ahead

- previewing the unit



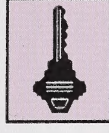
Exploring the Topic

- actively learning new concepts



Graphing Calculator

- using your graphing calculator



Key Idea

- flagging important ideas



Another View

- exploring different perspectives



Solutions

- correcting the activities



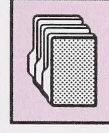
Extra Help

- providing additional study



Extensions

- going on with the topic



What You Have Learned

- summarizing what you have learned

Mathematics 10

Course Overview

Mathematics 10 contains 8 units. Beside each unit is a percentage that indicates what the unit is worth in relation to the rest of the course. The units and their percentages are listed below. You will be studying the unit that is shaded.

Unit 1 Number Systems	10%
Unit 2 Operations on Polynomials	14%
Unit 3 Equations and Inequalities	10%
Unit 4 Factoring Polynomials	13%
Unit 5 Coordinate Geometry	19%
Unit 6 Systems of Equations	10%
Unit 7 Trigonometry	11%
Unit 8 Statistics	13%
	100%

Unit Assessment

After completing the unit, you will be given a mark based totally on a unit assignment. This assignment will be found in the Assignment Booklet.

Unit Assignment - 100%

If you are working on a CML terminal, your teacher will determine what this assessment will be. It may be

Unit Assignment - 50%
Supervised Unit Test - 50%

Introduction to Number Systems

This unit covers topics dealing with number systems. Each topic contains explanations, examples, and activities to assist you in understanding number systems. If you find you are having difficulty with the explanations and the way the material is presented, there is a section called **Extra Help**. If you would like to extend your knowledge of the topic, there is a section called **Extensions**.

You can evaluate your understanding of each topic by working through the activities. Answers are found in the solutions in the **Appendix**. In several cases there is more than one way to do the question.

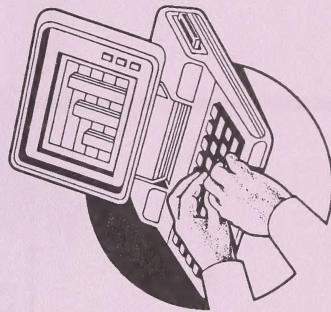
Unit 1 Number Systems

Contents at a Glance

Value	Number Systems	3
	What You Already Know	5
	Review	7
42%	Topic 1: Rational Number System	9
	• Introduction • What Lies Ahead • Exploring Topic 1 • Extra Help • Extensions	
32%	Topic 2: Irrational Number System	39
	• Introduction • What Lies Ahead • Exploring Topic 2 • Extra Help • Extensions	
26%	Topic 3: Relationships Among Number Systems	58
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Number Systems

Number systems is a branch of mathematics concerned with properties of numbers such as whole numbers, fractions, decimals, percents, signed numbers, and square roots. These properties have been objects of fascination and investigation for thousands of years. Interest in the natural numbers is as old as civilization. The topics in this unit relate to concepts that you studied previously. These concepts are important in future mathematics courses and in day-to-day life. Mathematicians have studied the natural numbers for hundreds of years and they are still learning more about them.



Unit 1

Number Systems

Topic 3
Relationships Among
Number Systems

Topic 2
Irrational Number
System

Topic 1
Rational Number
System



What You Already Know

Recall the following:

- adding, subtracting, multiplying, and dividing integers

Examples

$$-12 + 7 = -5$$

$$14 - 9 = 5$$

$$\begin{aligned} 23 - 31 &= 23 + (-31) \\ &= -8 \end{aligned}$$

$$6(-3) = -18$$

$$(-4)(-5) = 20$$

$$(-27) \div (3) = -9$$

$$\begin{aligned} \frac{35}{5} &= 35 \div 5 \\ &= 7 \end{aligned}$$

- adding, subtracting, multiplying, and dividing decimals

Examples

$$11.7 + 0.28 = 11.98$$

$$23.4 - 17.3 = 6.1$$

$$8.23 \times 6.5 = 53.495$$

$$123.7 \div 2.8 \approx 44.179$$

- rounding decimals

Examples

When 18.732 is rounded to the nearest hundredth, it is 18.73.

When -4.26 is rounded to the nearest tenth, it is -4.3.

When 0.96 is rounded to the nearest tenth, it is 1.0.

The symbol $\approx$ means
is approximately equal to,
and indicates that an answer
has been rounded.



- determining equivalent fractions

Examples

$$\frac{1}{5} = \frac{3}{15} \text{ because } 1 \times 15 = 5 \times 3$$

$$\frac{3}{8} \neq \frac{5}{16} \text{ because } 3 \times 16 \neq 8 \times 5$$

- finding the lowest common multiple (L.C.M.)

Examples

The L.C.M. of 6 and 5 is 30 because 30 is the smallest number that is divisible by both 6 and 5.

The L.C.M. of 8 and 12 is 24. Note that 48 and 72 are also common multiples of 8 and 12 but 24 is the lowest one possible.

- adding, subtracting, and multiplying fractions

Examples

$$2\frac{3}{8} + 1\frac{1}{8} = 3\frac{4}{8} \\ = 3\frac{1}{2}$$

$$1\frac{1}{4} + 2\frac{2}{3} = 1\frac{3}{12} + 2\frac{8}{12} \\ = 3\frac{11}{12}$$

$$8\frac{4}{5} - 2\frac{3}{5} = 6\frac{1}{5}$$

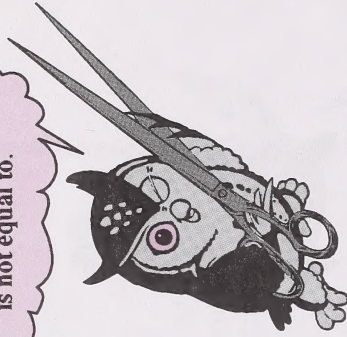
$$7\frac{3}{4} - 3\frac{1}{10} = 7\frac{15}{20} - 3\frac{2}{20} \\ = 4\frac{13}{20}$$

$$5\frac{1}{6} - 1\frac{5}{9} = 5\frac{3}{18} - 1\frac{10}{18} \\ = 4\frac{21}{18} - 1\frac{10}{18} \\ = 3\frac{11}{18}$$

$$\frac{4}{5} \times \frac{3}{7} = \frac{4 \times 3}{5 \times 7} \\ = \frac{12}{35}$$

$$\frac{7}{10} \times \frac{5}{6} = \frac{7 \times 5}{10 \times 6} \\ = \frac{7}{12}$$

The symbol $\neq$ means
is not equal to.



$$2\frac{1}{3} \times 3\frac{3}{5} = \frac{7}{3} \times \frac{18}{5}$$

$$= \frac{42}{5}$$

$$= 8\frac{2}{5}$$

$$\frac{3}{8} + \frac{1}{5} = \frac{3}{8} \times \frac{5}{1}$$

$$= \frac{15}{8}$$

$$= 1\frac{7}{8}$$

$$2\frac{2}{7} + 1\frac{1}{9} = \frac{16}{7} + \frac{10}{9}$$

$$= \frac{16}{7} \times \frac{9}{9}$$

$$= \frac{72}{35}$$

$$= 2\frac{2}{35}$$

Now that you have looked at material you studied previously, go to the **Review** to confirm your understanding of this material.



Review

Try the following review questions.

1. Add the following.

a. $-9 + 82$

b. $6 + (-21)$

c. $-5 + (-7)$

2. Subtract the following.

a. $-4 - 9$

b. $5 - 2$

c. $-7 - (-12)$

3. Multiply the following.

a. $3(-5)$

b. $-4(-8)$

c. $-15(5) + (-6)(2)$

4. Divide the following.

a. $72 \div 9$

b. $-63 \div 21$

c. $-125 \div (-5)$

5. Simplify.

a. $\frac{7(-12) - 7(22)}{3(5)(-7) + 7(13)}$

b. $15 \div (4 + 3 - 2) \times (8 - 11 + 2)$

6. Perform the following operations.

a. $5.002 + 0.07 + 18.6$

b. $4.893 - 3.02$

c. 0.5217×0.006

d. $756.82 \div 0.016$

7. Round each to the nearest one, nearest ten, nearest hundred, and nearest hundredth.

a. 16.825

b. 0.462

8. Show that the fractions are equivalent.

a. $\frac{15}{45}, \frac{5}{15}$

b. $\frac{7}{49}, \frac{3}{21}$

c. $\frac{36}{81}, \frac{8}{18}$

9. Find the lowest common multiple.

a. 5, 7

b. 4, 11, 6

10. Arrange $\frac{1}{2}, \frac{2}{5}, \frac{1}{6}$, and $\frac{1}{3}$ in ascending order.

11. Simplify. (Leave answers in lowest terms.)

a. $5\frac{1}{2} + 2\frac{1}{3}$

b. $3 - 1\frac{4}{7}$

c. $3\frac{1}{3} \times 1\frac{2}{7}$

d. $6\frac{1}{2} + \frac{13}{14}$



Now go to the **Review** solutions in the **Appendix**.

If you had trouble with the **Review**, you may wish to go to the **Extra Help** section, **Topic 1**.

Topic 1 Rational Number System



Introduction

Happy, joyous, delighted, and pleased are all words which express a similar idea. In math the same is true; many different forms can be used to express the same idea or quantity. In this topic you will learn to identify numbers and express them in different forms.

When you add or multiply two repeating decimals, will you get a repeating decimal? Test your math literacy.



What Lies Ahead

Throughout this topic you will learn to

1. identify repeating or terminating decimals as rational numbers
2. change fractions to decimals and percents
3. change terminating decimals to fractions and percents
4. recognize fractional equivalents of common repeating decimals (thirds, sixths, ninths)

Now that you know what to expect, turn the page to begin your study of rational number system.



Exploring Topic 1

Activity 1



Identify repeating or terminating decimals as rational numbers.

Rational numbers are numbers written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

In a baseball season Garry Soben was at bat 450 times and had 90 hits. The ratio, number of hits to number of times at bat expressed as a decimal equivalent, gives his batting average. Why do some sports, such as baseball and hockey, use decimals to keep records of their players? **Decimals are used for easy comparison.** The decimal equivalent for $\frac{90}{450}$ is 0.200.



Here are the statistics for Garry Soben and three other players. Which player has the best batting average?



Players	Times at Bat	Number of Hits	Ratio	Decimal
G. Soben	450	90	$\frac{90}{450} = \frac{1}{5}$	0.200
P. Coachman	310	93	$\frac{93}{310}$	
C. Cron	500	135		
D. Davis	156	39		

A batting average is shown using three decimal places where 0.200 would be read as a **batting average of 200**.

To help you answer the question, fill in the chart. Which numbers are easiest to compare?

By looking at the decimals, you can see that P. Coachman has the highest batting average with 300.



Use your calculator and convert $\frac{3}{8}$, $\frac{1}{3}$, $\frac{4}{11}$, and $\frac{5}{333}$ to decimals. Your answers should be $\frac{3}{8} = 0.375$, $\frac{1}{3} = 0.333\dots$, $\frac{4}{11} = 0.3636\dots$, and $\frac{5}{333} = 0.015015\dots$

The condensed form of $0.333\dots$ is $0.\overline{3}$ or $0.\dot{3}$, and the condensed form of $0.3636\dots$ is $0.\overline{36}$ or $0.\dot{36}$. Now write the short form for $0.015015\dots$. The answer is $0.\overline{015}$ or $0.\dot{015}$. Notice something special about 0.015 . The dots are placed over the first and last digits of the repeating block in the decimal. **Decimals that end like 0.375 are called terminating decimals. Decimals that repeat infinitely like $0.\overline{36}$ are called repeating decimals.**

Now try the questions to see if you understand these concepts.

Do questions 1 to 4. Question 5 is optional.

1. Differentiate between a repeating decimal and a terminating decimal.
2. Find the decimal equivalents for $\frac{1}{7}$, $\frac{3}{7}$, and $\frac{6}{7}$. These are unusual. Why?
3. a. In hockey, a goalkeeper's performance is judged by the number of goals scored on him per game. This is called goals against average. Explain how to determine goals against average.
b. A hockey game is sixty minutes long. If a goal keeper played for one period (twenty minutes) and allowed two goals to be scored on him, what is his goals against average for this game?

To convert a fraction to a decimal, divide the numerator by the denominator.

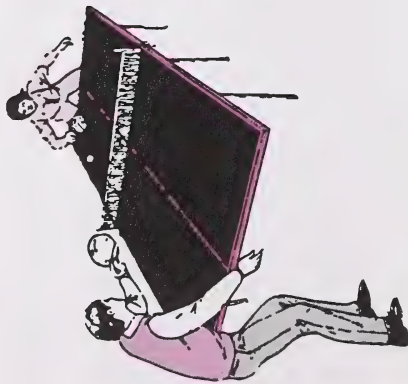
If the number of digits of the repeating block in the decimal are greater than 2, the dots are placed on the first and last digits.

Hint: Examine the number of digits in the repeating block.

4. a. Kim Choi and Darcy Lowe played a ping-pong match. The following is a table showing the smashing ability of each player. Complete the table, and express each answer as a decimal.

Players	Number of Successful Smashes	Number of Tries	Ratio $\frac{\text{successful smashes}}{\text{number of tries}}$
Kim Choi	5	8	
Darcy Lowe	8	11	

- b. Who is the better smasher, Kim or Darcy?



5. Which of the following are repeating decimals?

- 4.5
- $2.\overline{7}$
- $0.\overline{3217}$
- 0.68
- $1.04\overline{318}$



For solutions to Activity 1, turn to the **Appendix, Topic 1.**



Activity 2



Change fractions to decimals and percents.

You are familiar with fractions and you saw the importance of decimals in the previous section. You also saw that it was easier to compare decimals. Now look at percents.

While fractions and decimals play an important role in some areas, business persons compare their profits and losses by using percents. Fractions, decimals, and percents are related and they can be changed from one form to another.

Now study some examples to see how fractions, decimals, and percents are related.

Example 1

A farmer owns 200 hectares of land. He seeded 120 hectares. The remainder is in summer fallow.

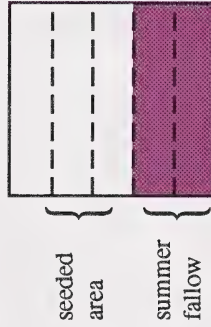
- Express the summer fallow as a fraction of the land the farmer owns.

Solution:

The number of hectares in summer fallow is equal to the total land less the seeded area.

$$200 \text{ ha} - 120 \text{ ha} = 80 \text{ ha}$$

The ratio, or number of hectares in summer fallow to total land, is $\frac{80 \text{ ha}}{200 \text{ ha}} = \frac{2}{5}$. Therefore, $\frac{2}{5}$ of the farmer's land is summer fallow.



- Express the fraction as a decimal.

Solution:

Divide 2 by 5. The decimal equivalent of $\frac{2}{5}$ is 0.4.



- Express the decimal as a percent.

Solution:

$$0.4 = \frac{4}{10}$$

Percent means **out of 100**. Rewrite $\frac{4}{10}$ as a fraction with a denominator of 100.

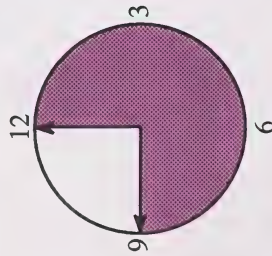
$$\begin{aligned} \frac{4}{10} &= \frac{?}{100} \\ &= \frac{40}{100} \end{aligned}$$

Therefore, $0.4 = 40\%$.

Example 2

Express 45 minutes out of an hour as a fraction, a decimal, and a percent.

Solution:



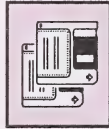
In one hour there are 60 minutes.

Therefore, $\frac{45 \text{ min}}{60 \text{ min}} = \frac{3}{4}$.

The decimal equivalent of $\frac{3}{4}$ is 0.75.

$$0.75 = \frac{75}{100}$$

As a percent, 0.75 is 75%.



For additional help, you may go to the Apple II diskette titled *Algebra Drill and Practice I*. Go to Part 4 called Percentages.

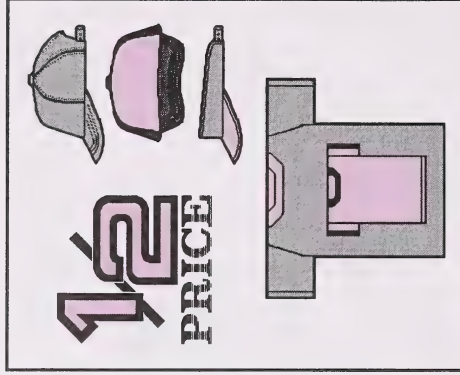
Now complete the questions.

Do questions 1, 2, 3, 4, 6, and 8. If you need more practice, do questions 5 and 7.

- Joseph is paid an annual salary of \$36 000. In 1987 he took two months' leave without pay. Find his loss in pay. Which of the three comparisons (fractions, decimals, or percents) is most appropriate for this calculation? Explain why you made this choice.
- Businesses such as clothing stores usually work their sale items as a percent. Why are decimals not used?

¹ *Algebra Drill and Practice I* is a title of Conduit.

Percent means **per hundred**.
Note that 40% can be written as $\frac{40}{100}$ or 40:100.



3. In a football game a quarterback completed 9 out of 11 passes. Express this ratio as a fraction, a decimal, and a percent. The commentator avoids the use of one of these. Identify it and state why this statistic is not practical in a football game.

4. The Gallup poll is always given as a percent to show each political party's standing. Why are fractions or decimals not used?

5. For each of the following state the most appropriate form of comparison.

- a. player's batting average
- b. storekeeper's discount
- c. shares in stockmarket
- d. goalkeeper's goals against average
- e. auto dealer's profits
- f. causes of fatal auto accidents
- g. amount of land seeded

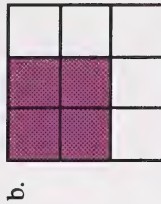
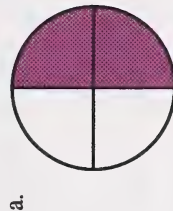
6. The diagram represents a rectangular piece of paper divided into equal squares. A square in each corner is cut away and the paper is folded into a tray. What fraction of the paper is used for the bottom of the tray? Write this fraction as a percent.



7. Complete the chart.

Fraction	Decimal	Percent
$\frac{7}{8}$		
		75.5%
	0.7	

8. For the shaded part of each diagram state the fraction, decimal, and percent.



For solutions to Activity 2, turn to the Appendix, Topic 1.

Activity 3



Change terminating decimals to fractions and percents.

You can change decimals to fractions by using the appropriate place value. Recall from your previous study of math that the place value of each digit is a number. Study the number 849.731.

8	4	9	.	7	3	1
hundreds	tens	ones (units)		tenths	hundredths	thousandths

This decimal can be converted to a fraction using this place value idea. **Since the last place value is thousandths, the denominator will be 1000.** The numerator is 849 731.

$$\frac{849\,731}{1000}$$

Now look at some examples, and see how you can relate a decimal to its fraction and also to a percent.

Example 3

This graduated beaker can hold up to 1 L of water. It now contains 125 mL of water.

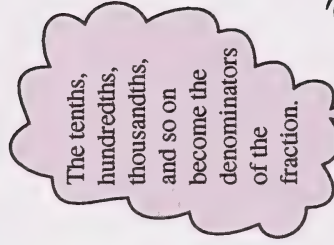


- Express the amount of water in the beaker as a fraction of the total liquid it can hold.

Solution:

$$125\text{ mL} = 0.125\text{ L}$$

Assign the place value for each digit in 0.125.



0 . 1 2 5
units tenths hundredths thousandths

To change 0.125 to a fraction, write the numerator as 125. The denominator is 1000 because the 5 is in the thousandths place.

The fraction is $\frac{125}{1000}$.

- Express the fraction as a percent.

Solution:

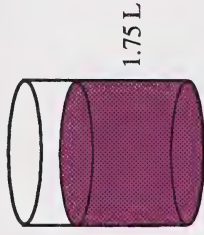
To convert $\frac{125}{1000}$ to a percent, you need an equivalent fraction with a denominator of 100. Divide the numerator and denominator by 10. This gives you $\frac{12.5}{100}$.

You have satisfied the definition of percents.

$$\begin{aligned}\frac{125}{1000} &= \frac{125 \div 10}{1000 \div 10} \\ &= \frac{12.5}{100} \\ &= 12.5\%\end{aligned}$$

Example 4

- A two-litre jar contains 1.75 litres of syrup. What fraction of the jar contains syrup?



Solution:

The two-litre jar contains 1.75 L of syrup. Therefore, the amount of syrup in the jar is $\frac{1.75 \text{ L}}{2 \text{ L}}$ or $\frac{175}{200}$ or $\frac{7}{8}$.

- Write the fraction as a percent. Use $\frac{175}{200}$.

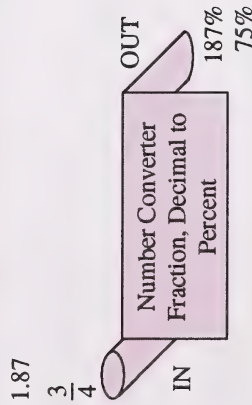
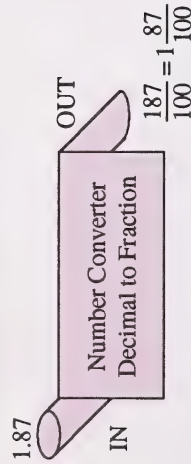
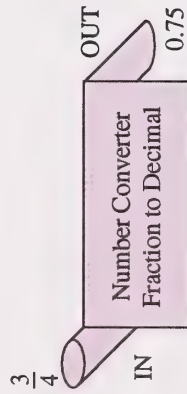
Solution:

To convert $\frac{175}{200}$ to a percent you need a fraction with denominator of 100.

$$\begin{aligned}\frac{175}{200} &= \frac{175 \div 2}{200 \div 2} \\ &= \frac{87.5}{100} \\ &= 87.5\%\end{aligned}$$

To get a denominator of 100, divide both the numerator and the denominator by 2.

Similarly you can change any decimal to a fraction or percent.



Now look at a more complex number.

Example 5

- Change 348.257 to a fraction.

Solution:

The seven is in the thousandths place; thus, the denominator for the resulting fraction is 1000. Therefore, $348.257 = \frac{348\,257}{1000}$.

- Change the fraction to a percent.

Solution:

You should be familiar with the definition of percents after doing Example 3 and Example 4. You have to write $\frac{348\,257}{1000}$ as a fraction with a denominator of 100.

If you divide the numerator and denominator of $\frac{348\,257}{1000}$ by 10, you will get the following.

$$\frac{348\,257}{1000} + \frac{10}{10} = \frac{348\,257 + 10}{1000 + 10} = \frac{34\,825.7}{100}$$

The percent rule is satisfied. Therefore, $\frac{348\,257}{1000} = 34\,825.7\%$.

Now do the following questions.

1. An orange is cut into eight equal pieces. Which of the three comparisons (decimals, fractions, or percents) best describes a part of the orange? Why?
2. Convert each to a fraction and a percent.
 - a. 0.57
 - b. 16.4
 - c. 128.426



For solutions to **Activity 3**, turn to the **Appendix, Topic 1**.

Activity 4



Recognize fractional equivalents of common repeating decimals (thirds, sixths, ninths).

Not all fractions convert to terminating decimals. Some fractions convert to repeating decimals. Now look at an example that involves repeating decimals.

Note: To change fractions to decimals, divide the top number by the bottom number.

Example 6

Use a calculator to help change the following fractions to decimals.

$$\bullet \frac{1}{3}$$

Solution:

	Enter	Display
	1	1
	$\div$	1
	3	3
	$=$	0.3333333333

Therefore, $\frac{1}{3} = 0.\overline{3}$.

$$\bullet \frac{1}{6}$$

Solution:

	Enter	Display
	1	1
	$\div$	1
	6	6
	$=$	0.1666666666

Therefore, $\frac{1}{6} = 0.\overline{16}$.

Use a calculator to change the following fractions to decimals.

1. $\frac{2}{3} =$ _____

2. $\frac{4}{3} =$ _____

3. $\frac{1}{9} =$ _____

4. $\frac{2}{9} =$ _____

5. $\frac{3}{9} = \frac{1}{3} =$ _____

6. $\frac{2}{6} =$ _____

7. $\frac{3}{6} =$ _____

8. $\frac{4}{6} =$ _____



For solutions to Activity 4, turn to the Appendix,
Topic 1.

You will find that when fractions convert to repeating decimals, the repeating digits have a definite pattern. Try some on your own.

With some memory work and practice you should be able to recognize the fractions for $0.\overline{3}$, $0.\overline{6}$, and $0.\overline{1}$. They are $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{1}{9}$ respectively. Give some thought to $33\frac{1}{3}\% = \frac{1}{3}$, $66\frac{2}{3}\% = \frac{2}{3}$, and $11\frac{1}{9}\% = \frac{1}{9}$.

It is very convenient to convert fractions to repeating decimals using a calculator. How do you change repeating decimals to fractions? Using a calculator is not practical since you have to make the conversions by trial and error.

If you are interested in an algebraic method to convert repeating decimals to fractions, see Converting Repeating Decimals to Fractions in the **Extensions, Topic 1**.



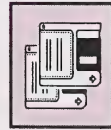
If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

} You may decide to do both.



Extra Help

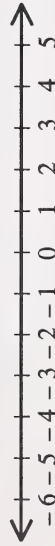


You may go to the Apple II diskette titled *Algebra Drill and Practice I*¹. Go to Part 5 called Signed Numbers and work through the program in place of doing the Integers section of the **Extra Help**.

When you have completed the software program, go to the Operations with Decimals section of **Extra Help**.

Integers

Signed numbers called **integers** are used throughout mathematics. Integers include all positive and negative whole numbers and zero.



See how much you recall about integer rules by looking at the following example.

Example 7

Do the following operations.

• $8 - 13$

Solution:

$$8 - 13 = -5$$

• $(-20)(+15)$

Solution:

$$(-20)(+15) = -300$$

• $(-2)(-5) + (3)(-4)$

Solution:

$$\begin{aligned} (-2)(-5) + (3)(-4) &= 10 + (-12) \\ &= -2 \end{aligned}$$

¹ *Algebra Drill and Practice I* is a title of Conduit.

$$\bullet 5[-12 + 3 + (4 - 8) + (6 - 8)]$$

Solution:

$$\begin{aligned} 5[-12 + 3 + (4 - 8) + (6 - 8)] &= 5[-4 + (-4) + (-2)] \\ &= 5[-4 + 2] \\ &= 5[-2] \\ &= -10 \end{aligned}$$

Adding Integers

When adding negative integers and positive integers, subtract the two numbers and use the sign of the number with the larger face value. Look at this example.

Example 8

Evaluate $-9 + 2$.

Solution:

You may think of \$9 in debt and \$2 in profit. The result is \$7 in debt. Therefore, $-9 + 2 = -7$.

When you add negative integers to negative integers or positive integers to positive integers, add the numbers and use the common sign.

Watch the signs.

Remember the rules for adding and subtracting like and unlike signs.



Example 9

Do the following operations.

- $-9 + (-2)$

Solution:

\$9 in debt and \$2 more in debt is equal to \$11 in debt.

Therefore, $-9 + (-2) = -11$.

- $+3 + (+9)$

Solution:

$$+3 + (+9) = 3 + 9 \\ = +12$$

Subtracting Integers

To subtract an integer, add its opposite (its additive inverse). Here are two examples of opposites:

- The opposite or additive inverse of 7 is -7 .
- The opposite or additive inverse of -5 is $+5$.

The next example shows you how to subtract integers.

Example 10

Do the following operations.

- $5 - (-13)$

Solution:

When a debt of \$13 is taken away, it is no longer a debt and thus becomes a positive quantity.

Since $-(-13) = +13$, then $5 - (-13) = 5 + 13$
 $= 18$

- $11 - 7$

Solution:

A debt of \$7 and a profit of \$11 will leave a profit of \$4. Therefore, $11 - 7 = 4$.

You must also note the following:

- $10 - 7 = 3$ or $10 + (-7) = 3$
- $-3 - 4 = -7$ or $-3 + (-4) = -7$

As long as you add the opposites, subtraction can be rewritten as an addition question.

Multiplying Integers

The product of two integers with unlike signs is negative.

$$\begin{aligned}(-) \times (+) &= (-) \\ (+) \times (-) &= (-)\end{aligned}$$

The product of two integers with like signs is positive.

$$\begin{aligned}(+) \times (+) &= (+) \\ (-) \times (-) &= (+)\end{aligned}$$

Check these rules in the following.

- $-6 \times 10 = -60$
- $-4 \times (-7) = 28$

Dividing Integers

The quotient of two integers with like signs is positive.

$$\begin{aligned}(-) \div (-) &= (+) \\ (+) \div (+) &= (+)\end{aligned}$$

The quotient of two integers with unlike signs is negative.

$$\begin{aligned}(-) \div (+) &= (-) \\ (+) \div (-) &= (-)\end{aligned}$$

These rules are shown in the examples that follow.

- $-12 \div (-4) = 3$ (like signs)
- $12 \div 4 = 3$ (like signs)
- $-12 \div 4 = -3$ (unlike signs)
- $12 \div (-4) = -3$ (unlike signs)

Order of Operations

In mathematics there are rules to follow when performing several numerical calculations. Operations must be done in a special order.

The order is as follows:

1. Do the operations inside the brackets. Do the inner brackets first and then the outer brackets.
2. Simplify any exponents.
3. Do the division and multiplication as they occur from left to right.
4. Do the addition and subtraction as they occur from left to right.

The next examples demonstrate these rules.

Use **BEDMAS** to help you remember the order.

B = Brackets
E = Exponents
D = Division or
M = Multiplication
A = Addition or
S = Subtraction

Example 11

Calculate $[(3+5) + (2 \times 2)] - (6-4)$.

Solution:

$$\begin{aligned} [(3+5) + (2 \times 2)] - (6-4) &= [8+4] - 2 \\ &= 2-2 \\ &= 0 \end{aligned}$$

Example 12

Calculate $\frac{14 \times 9 + 3}{3 \times 5 + 6}$.

Solution:

$$\begin{aligned} \frac{14 \times 9 + 3}{3 \times 5 + 6} &= \frac{126 + 3}{15 + 6} \\ &= \frac{42}{21} \\ &= 2 \end{aligned}$$

Example 13

Calculate $\frac{3 \times (9-2) + 8 \div 4 + 2}{6 \times (11-8) - (8+5)}$.

Solution:

$$\begin{aligned} \frac{3 \times (9-2) + 8 \div 4 + 2}{6 \times (11-8) - (8+5)} &= \frac{3 \times 7 + 2 + 2}{6 \times 3 - 13} \\ &= \frac{21 + 2 + 2}{18 - 13} \\ &= \frac{25}{5} \\ &= 5 \end{aligned}$$

Try the questions that follow, and see if you understand the concepts.

Do every second question in each section. You may complete the others if you are having difficulty. No calculators are allowed except where mentioned.

1. Add the following.

a. $-9+8$ b. $6+(-21)$

c. $-3+(-5)$ d. $8+7$

e. $7+(-13)+(-26)+42$ f. $-81+58+26+(-38)$

2. Subtract the following.

- a. $-5 - (-12)$ b. $-8 - 6$
 c. $9 - 3$ d. $7 - (-8)$
 e. $13 - (-9)$ f. $7 - (-3) - 6 - (-9)$

3. Multiply the following.

- a. $11(-6)$ b. $-9(-4)$
 c. $-10(10)$ d. $7(5)$
 e. $-5(9)(-2)$ f. $-12(4)(-3)(2)$

4. Divide the following.

- a. $65 \div 13$ b. $-15 \div (-3)$
 c. $\frac{-27}{9}$ d. $\frac{56}{-7}$

5. Use a calculator for the following.



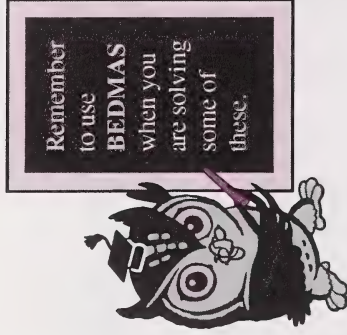
- a. $\frac{273 - 2765}{89}$
 b. $\frac{7(12) - 7(22)}{3(5)(-7) + 7(13)}$

6. Simplify the following.

- a. $12 \div 4 + 8 \div 2$
 b. $(15 - 9) \div (24 \div 8)$
 c. $\frac{75 \div 5 \div 3}{2 \times 2 \times 2 - 7}$
 d. $15 \div (4 + 3 - 2) \times (8 - 11 + 2)$
 e. $\frac{16 \div (9 - 5) \times 2}{19 - (14 + 3)}$
 f. $\frac{[(3 + 2) \times (5 - 3)] + (12 - 10)}{[(9 - 5) \div (11 - 9)] + (9 - 5)}$



For solutions to **Extra Help**,
 turn to the **Appendix, Topic 1**.



Operations with Decimals

Another important skill is the ability to calculate using decimals.

How many of these decimal questions can you do?

- $0.503 + 0.9093$
- $12.6 - 11.931$
- -5.02×0.2
- $(-0.0064) + (0.004)$

Did you get 1.4123, 0.669, -1.004 , and -1.6 respectively?

Adding and Subtracting Decimals

Any two decimals can be added or subtracted. Simply line up the decimal points vertically, perform the operations as you would with whole numbers, then place the decimal point in the answer directly below the decimal points in the question. Study the following examples.

Example 14

Calculate $3.27 + 5 + 6.9$.

Solution:

$$\begin{array}{r} 3.27 \\ 5.00 \\ + 6.90 \\ \hline 15.17 \end{array}$$

Example 15

Calculate $4.93 - 3.126$.

Solution:

$$\begin{array}{r} 4.930 \\ - 3.126 \\ \hline 1.804 \end{array}$$

Multiplying Decimals

Multiply decimals as you would multiply whole numbers (ignoring the decimal). The number of digits that should appear after the decimal point in the product (answer) is the sum of the number of digits appearing after the decimal point in each of the numbers being multiplied. Examine the next example.

When adding or subtracting decimals, you must put in extra zeros to fill up the spaces set out by the longest number.



Example 16

Calculate 1.21×9.2 .

Solution:

$$\begin{array}{r} 1.21 \quad \text{(two digits after the decimal point)} \\ \times 9.2 \quad \text{(one digit after the decimal point)} \\ \hline 242 \\ 10890 \\ \hline 11.132 \quad \text{(three digits after the decimal point)} \end{array}$$

Dividing Decimals

To divide decimals, move the decimal point to make the divisor a whole number. Then move the decimal in the dividend the same number of places. Place the decimal point in the quotient directly above the new decimal point in the dividend and divide. Look at the following example.

Example 17

Calculate $3.831 \div 0.3$.

Solution:

$$3.831 \div 0.3 \rightarrow 38.31 \div 3 \rightarrow 3 \overline{)38.31} \begin{array}{r} 12.77 \\ 38.31 \end{array}$$

The answer is 12.77.

Try some of the following on your own. These question are to be done without the use of a calculator.

Complete numbers 7, 8, 11, 12, and 14 from the following exercise. You may do the others if you need more practice.

7. $3.002 + 45.73 + 12.8$

8. $8.26 - 4.3$

9. $15 - 9.37$

10. 32.8×6.7

11. 0.721×0.05

12. $100.8 \div 0.72$

13. $3.4 + 0.9 + 3$

14. $38.5 + (5.5 + 11)$



For solutions to **Extra Help**, turn to the **Appendix, Topic 1**.

Recall:

dividend $\div$ divisor = quotient

or

$$\begin{array}{r} \text{quotient} \\ \text{divisor} \overline{) \text{dividend}} \end{array}$$

In Example 17, 3.831 is the dividend, 0.3 is the divisor, 38.31 is the new dividend, and 3 is the new divisor.

Rounding Numbers

When working with decimals it is often necessary to round numbers.

The following examples review rounding rules for decimals. Try them yourself to test your rounding skills.

- Round 1.0445 to the nearest hundredth.
The answer is 1.04.

- Round 2.996 to the nearest tenth.
The answer is 3.0.

You will note that this answer is written as 3.0 and not 3 since the tenths place must be shown.

If you are asked to round a number, look at the reference digit (the number immediately following the chosen place value).

When the reference digit is less than 5, do not change the previous digit.

When the reference digit is 5 or more, increase the previous digit by one.

Now study the following to see how this process works when applied to any number. Here, 3.4725 is used.

When 3.4725 is rounded to the nearest one, it is 3.
(The reference digit is 4.)

When 3.4725 is rounded to the nearest tenth, it is 3.5.
(The reference digit is 7.)

When 3.4725 is rounded to the nearest hundredth, it is 3.47.
(The reference digit is 2.)

When 3.4725 is rounded to the nearest thousandth, it is 3.473.
(The reference digit is 5.)

Now it is your turn. Try the following question.

15. Round each to the nearest ten, nearest one, nearest tenth, and nearest hundredth.

Number	Nearest			
	ten	one	tenth	hundredth
13.465				
2.9745				
8.905				
384.782				
7.398				
0.902				



For solutions to **Extra Help**, turn to the **Appendix, Topic 1.**

Equivalent Fractions

Fractions occur frequently in mathematics.

What do you recall from previously-learned skills with fractions?

Are the fraction pairs shown here equivalent?

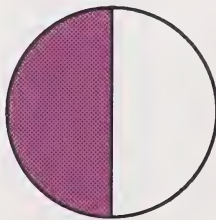
$$\frac{2}{3} = \frac{3}{4}$$

$$\frac{10}{12} = \frac{15}{18}$$

$$\frac{3}{7} = \frac{6}{10}$$

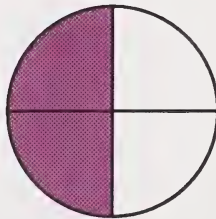
The first and third pairs are not equivalent.

Equivalent fractions are fractions that represent the same amount. They are fractions of equal value. Examine the following examples.



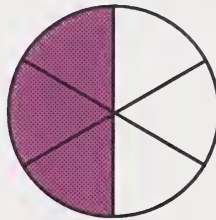
$$\frac{1}{2}$$

=



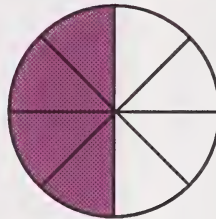
$$\frac{2}{4}$$

=



$$\frac{3}{6}$$

=



$$\frac{4}{8}$$

To generate fractions equivalent to a given fraction, one method is to multiply the given fraction by other fractions equivalent to one.



Example 18

Express $\frac{1}{2}$ in sixths, tenths, and quarters.

Solution:

$$\frac{1}{2} = \frac{1}{2} \times \frac{3}{3}$$

$$= \frac{3}{6}$$

$$\frac{1}{2} = \frac{1}{2} \times \frac{5}{5}$$

$$= \frac{5}{10}$$

$$\frac{1}{2} = \frac{1}{2} \times \frac{2}{2}$$

$$= \frac{2}{4}$$

Therefore, $\frac{1}{2}$, $\frac{3}{6}$, $\frac{5}{10}$, and $\frac{2}{4}$ are equivalent fractions.

In order to determine if a pair of fractions is equivalent, one method is to use the following fact:

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } ad = bc.$$

This is called cross-multiplying where ad and bc are cross products.

Example 19

Is $\frac{1}{2} = \frac{3}{6}$?

Solution:

$$6 \times 1 = 2 \times 3$$

$$6 = 6 \text{ (true)}$$

Therefore, $\frac{1}{2} = \frac{3}{6}$ is a true statement.

Example 20

Is $\frac{3}{4} = \frac{2}{3}$?

Solution:

$$3 \times 3 = 2 \times 4$$

$$9 = 8 \text{ (false)}$$

Therefore, $\frac{3}{4} \neq \frac{2}{3}$; $\frac{3}{4}$ and $\frac{2}{3}$ are not equivalent fractions.

Recall: The symbol $\neq$ means is not equal to.



It is time to practise. Do any three problems from question 16 and two problems from question 17.

16. Show that the fractions are equivalent.

a. $\frac{8}{12}, \frac{10}{15}$

b. $\frac{4}{20}, \frac{3}{15}$

c. $\frac{18}{24}, \frac{12}{16}$

d. $\frac{6}{15}, \frac{16}{40}$

e. $\frac{4}{14}, \frac{6}{21}$

f. $\frac{5}{20}, \frac{4}{16}$

17. Write at least three fractions equivalent to each of the following.

a. $\frac{3}{5}$

b. $\frac{7}{10}$

c. $\frac{20}{25}$



For solutions to **Extra Help**, turn to the **Appendix, Topic 1**.

Lowest Common Multiple

Realizing that fractions have more than one equivalent form leads to work with common multiples and common denominators. Can you find the lowest common multiples for the following?

- 1, 2, 4, 6
The answer is 12.

- 2, 3, 4, 5
The answer is 60.

Recall that the lowest common multiple of two or more numbers is the smallest natural number divisible by each of the numbers. To find the lowest common multiple, consider the multiples of each number. The next example should help clarify this.

Example 21

Find the lowest common multiple of 4 and 8.

Solution:

Number	Multiples	Common Multiples
4	4, 8, 12, 16, 20, 24, 28, 32, ...	8, 16, 24, 32, ...
8	8, 16, 24, 32, 40, 48, ...	

Therefore, the lowest common multiple (L.C.M.) for 4 and 8 is 8.

A lowest common multiple can also be used as the lowest common denominator (L.C.D.) when adding or subtracting fractions.

Example 22

Find the lowest common multiple for the denominators in

$$\frac{1}{2}, \frac{2}{3}, \text{ and } \frac{1}{4}.$$

Solution:

Number	Multiples	Common Multiples
2	2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, ...	12, 24, ...
3	3, 6, 9, 12, 15, 18, 21, 24, 27, ...	
4	4, 8, 12, 16, 20, 24, 28, 32, ...	

The lowest common multiple (L.C.M.) for 2, 3, and 4 is 12.

Do the following on your own.

18. Find the lowest common multiple (L.C.M.) for each of the following.

a. 2, 3, 6

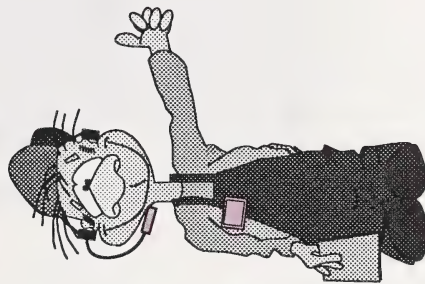
b. 3, 7

c. 12, 18

d. 4, 5, 6



For solutions to **Extra Help**, turn to the **Appendix, Topic 1**.



Operations with Fractions

Operations with fractions are very important. Review this topic by looking at some examples.

Example 23

Simplify $2\frac{1}{2} + 4\frac{3}{4} + 2\frac{1}{3}$.

Solution:

Change each fraction to an improper fraction.

$$2\frac{1}{2} = \frac{5}{2}$$

$$4\frac{3}{4} = \frac{19}{4}$$

$$2\frac{1}{3} = \frac{7}{3}$$

$$2\frac{1}{2} + 4\frac{3}{4} + 2\frac{1}{3} = \frac{5}{2} + \frac{19}{4} + \frac{7}{3}$$

Find the lowest common denominator for the denominators 2, 4, and 3. The L.C.D. is 12.

Change each fraction to its equivalent with a denominator of 12.

$$\frac{5}{2} + \frac{19}{4} + \frac{7}{3} = \frac{30}{12} + \frac{57}{12} + \frac{28}{12}$$

Add the numerators and put the sum over the common denominator 12.

$$\frac{30}{12} + \frac{57}{12} + \frac{28}{12} = \frac{115}{12}$$

This can be changed to a mixed number.

$$\frac{115}{12} = 9\frac{7}{12}$$

At this point further simplification may be needed. This is not the case here.

Example 24

Solve $10 - 3\frac{5}{9}$.

Solution:

$$10 - 3\frac{5}{9} = \frac{10}{1} - \frac{32}{9} \quad (\text{improper fractions})$$

The lowest common denominator is 9.

$$\frac{10}{1} - \frac{32}{9} = \frac{90}{9} - \frac{32}{9} \quad (\text{equivalent fractions with a denominator of 9})$$

$$= \frac{58}{9} \quad (\text{Subtract the numerators.})$$

$$\text{or } = 6\frac{4}{9} \quad (\text{Change to a mixed number.})$$

Hint: to change a mixed fraction to an improper fraction, multiply the whole number by the denominator of the fraction and add the numerator. Then, express this value over the denominator.

$$\begin{aligned} \text{For example, } 2\frac{2}{3} &= \frac{2 \times 3 + 2}{3} \\ &= \frac{8}{3} \end{aligned}$$

Hint: To change an improper fraction to a mixed fraction, divide the numerator by the denominator.

Example 25

- Solve $1\frac{3}{5} \times \frac{3}{4}$.

Solution:

$$\begin{aligned} 1\frac{3}{5} \times \frac{3}{4} &= \frac{8}{5} \times \frac{3}{4} && \text{(improper fractions)} \\ &= \frac{\overset{2}{8}}{5} \times \frac{\overset{3}{3}}{\underset{1}{4}} && \text{(Divide by the common factor of 4.)} \\ &= \frac{6}{5} && \text{(Multiply the numerators together, and multiply the denominators together.)} \\ &= 1\frac{1}{5} \end{aligned}$$

- Solve $3\frac{1}{3} \times 1\frac{1}{8}$.

Solution:

$$\begin{aligned} 3\frac{1}{3} \times 1\frac{1}{8} &= \frac{10}{3} \times \frac{9}{8} && \text{(improper fractions)} \\ &= \frac{\overset{5}{10}}{\underset{1}{3}} \times \frac{\overset{3}{9}}{\underset{4}{8}} && \text{(Divide by the common factors of 3 and 2.)} \\ &= \frac{15}{4} && \text{(Multiply the numerators together, and multiply the denominators together.)} \\ &= 3\frac{3}{4} \end{aligned}$$

Example 26

- Solve $4\frac{2}{5} \div 3\frac{1}{2}$.

Solution:

$$\begin{aligned} 4\frac{2}{5} \div 3\frac{1}{2} &= \frac{22}{5} \div \frac{7}{2} && \text{(improper fractions)} \\ &= \frac{22}{5} \times \frac{2}{7} && \text{(To divide invert the fraction and multiply.)} \\ &= \frac{44}{35} && \text{(Multiply the numerators together, and multiply the denominators together.)} \\ &= 1\frac{9}{35} \end{aligned}$$

Now try some questions for more practice.

Complete questions a, d, f, h, j, k, m, and n. Do the other questions if you need more practice.

Hint: When multiplying fractions, make your work easier by checking to see if any of the numerators or denominators have a common factor. If you find one, divide by it before multiplying.

When dividing by a fraction, multiply the dividend by the reciprocal of the divisor. The dividend is the first number and the divisor is the second number in this kind of problem:

$$\begin{array}{ccc} & \text{divisor} & \\ & \uparrow & \\ \text{for example, } 4\frac{2}{5} \div 3\frac{1}{2} = & & \\ & \downarrow & \\ & \text{dividend} & \end{array}$$

19. Simplify the following. Answers must be in lowest form.

a. $\frac{1}{3} + \frac{2}{5}$

b. $\frac{3}{8} + \frac{3}{6}$

c. $3\frac{1}{3} + 2\frac{1}{2}$

d. $2\frac{2}{3} + 5\frac{3}{4}$

e. $\frac{1}{2} - \frac{1}{3}$

f. $\frac{3}{4} - \frac{2}{3}$

g. $2 - 1\frac{5}{7}$

h. $5\frac{1}{2} - 2\frac{3}{4}$

i. $\frac{3}{5} \times \frac{7}{9}$

j. $\frac{2}{9} \times \frac{6}{5}$

k. $2\frac{1}{2} \times \frac{5}{8}$

l. $2\frac{1}{4} \times 1\frac{1}{5}$

m. $\frac{1}{2} \div \frac{4}{3}$

n. $\frac{4}{5} \div \frac{8}{5}$

o. $6\frac{1}{4} \div 2\frac{1}{2}$

p. $5\frac{1}{2} \div \frac{11}{12}$



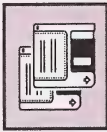
For solutions to **Extra Help**, turn to the **Appendix, Topic 1**.

Extensions

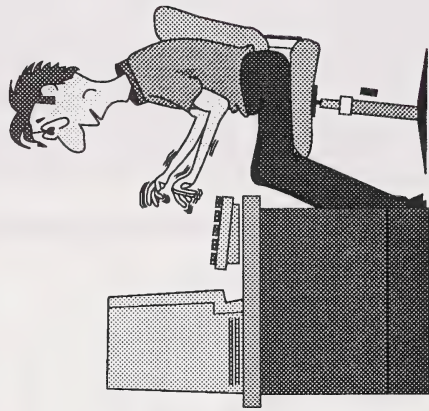


An Application of Rounding

You already know that working with decimals involves rounding. A practical application of rounding involves the reading of the scales. This can be any scale from a simple ruler to specialized instruments.



You may go to the Apple II diskette titled *Mathematics for Science, Series 1: Measurement*¹ for an application of rounding with reading scales.



¹ *Mathematics for Science, Series 1: Measurement* is a title of Merian Scientific Ltd.

Converting Repeating Decimals to Fractions

Example 27

Find an equivalent reduced fraction for $0.\overline{26}$.

Solution:

Let x be the fraction you are seeking.

Therefore, $x = 0.\overline{26}$. One digit is repeating; so multiply both sides of the equation by 10.

$$x = 0.\overline{26}$$

$$10x = 2.\overline{26}$$

$$10x = 2.\overline{26} \quad \text{(Fill this space with the repeating digit 6 for convenience of subtraction.)}$$

$$- x = 0.\overline{26}$$

$$9x = 2.4 \quad \text{(Subtract the two equations.)}$$

$$x = \frac{2.4}{9} \quad \text{(Multiply the fraction by } \frac{10}{10} \text{ to remove the decimal point.)}$$

$$= \frac{24}{90} \quad \text{(Reduce the fraction } \frac{24}{90} \text{ by dividing 24 and 90 by the G.C.F. which is 6.)}$$

$$= \frac{4}{15}$$



Use your calculator and check to see if $\frac{4}{15} = 0.\overline{26}$ or $0.26666 \dots$

Now work through a more difficult example.

Example 28

Write $2.\overline{6132}$ as a fraction.

Solution:

Let x be the fraction you are seeking.

Therefore, $x = 2.\overline{6132}$. Three digits are repeating; thus, multiply both sides of the equation by 1000. This gives you $1000x = 2613.\overline{2}$.

$$1000x = 2613.\overline{2} \quad \text{(Fill this space with the repeating block 132 for convenience of subtraction.)}$$

$$- x = 2.\overline{6132}$$

$$999x = 2610.6 \quad \text{(Multiply the fraction by } \frac{10}{10} \text{ to remove the decimal.)}$$

$$x = \frac{2610.6}{999} \quad \text{(Reduce the fraction. Divide by the G.C.F. of 26106 and 9990 which is 6.)}$$

$$= \frac{26106}{9990}$$

$$= \frac{4351}{1665}$$

$$= 2\frac{1021}{1665} \quad \text{(Simplify.)}$$



Check the answer with your calculator.

Note that $0.\overline{26}$ means the same as $0.2\overline{6}$. The dot and the bar mean the same.

The size of the repeating block determines what the multiplier will be. For $0.2\overline{6}$, multiply by 10 because there is one digit in the repeating block.

Now do the following questions to check your understanding.

1. The product of $0.\dot{6} \times 0.8\dot{3}$ is a repeating decimal. Explain how you establish this. Find the repeating-decimal product.
2. Convert each of the following to an equivalent fraction in simplest form.

a. $3.\dot{2}\dot{6}$

b. $0.02\dot{7}$

c. $11.\dot{2}3\dot{5}$

You may try the following questions if you need more practice.

3. a. $0.\overline{15} + 0.\overline{67}$

b. $0.\overline{73} - 0.\overline{51}$

4. a. $0.\overline{2} \times 1.\overline{3}$

b. $0.75 \times 2.\overline{6}$



For solutions to **Extensions**, turn to the **Appendix, Topic 1**.

Topic 2 Irrational Number System



Introduction

In the last topic rational numbers were discussed. However, not all numbers are rational. Numbers like π and 30.981 624 675 228 ... do not fit the idea of being rational; so they are called irrational. In this topic you will look at irrational numbers.



What Lies Ahead

Throughout this topic you will learn to

1. recognize irrational numbers as infinite, non-repeating decimals
2. find approximations for square roots of non-perfect squares
3. evaluate square roots and write the square roots of negative integers as imaginary numbers
4. recognize π as a special irrational number

Now that you know what to expect, turn the page to begin your study of the irrational number system.



Exploring Topic 2



Now find the decimal representation of $\sqrt{3}$ and $\sqrt{5}$. Do you see a pattern in these decimals? If you examine these square roots closely, you will notice that none of them have a definite pattern in the decimal section. **Numbers that do not repeat or terminate are called irrational numbers.**

The set of irrational numbers is represented by $\bar{Q}$.

Activity 1



Recognize irrational numbers as infinite, non-repeating decimals.



You will recall from **Topic 1** in this unit that decimals either repeat or terminate. Now you will look at numbers that do not repeat or terminate. One such commonly used number is $\sqrt{2}$. Use your calculator to find the decimal equivalent of $\sqrt{2}$.

Enter	Display
2	2
	1.414213562

The number of digits in your answer depends on your calculator. Do you see a pattern in this number?

An irrational number is a number that cannot be written in the form $\frac{a}{b}$, where a and $b \in I$ and $b \neq 0$.

Note that a and $b \in I$ means that a and b are integers.



Example 1

Why is $\sqrt{7}$ an irrational number?

Solution:

Try to use $\frac{a}{b}$ to find $\sqrt{7}$.

$$\sqrt{7} = 2.645\,751\,311\dots$$

Choose fractions so that their decimal representation is as close as possible to $2.645\,751\,311\dots$

One such fraction is $\frac{512}{193}$ which is equal to $2.652\,849\,741\dots$ This is a little too high.

The fraction $\frac{512}{194}$ is equal to $2.639\,175\,258\dots$ This is a little too low.

You can find fractions that will give you decimals that are closer to $2.645\,751\,311\dots$ but you cannot find an exact fractional equivalent. Therefore, you cannot express $\sqrt{7}$ in the form $\frac{a}{b}$.

In the decimal representation of $\sqrt{7}$, there is no digit or sequence of digits that repeats.

Answer the questions that follow.

1. State whether the number is rational or irrational. Give a reason for your answer.

- a. $\sqrt{87}$ b. $\sqrt{169}$
c. $6.312\,418\,526\dots$ d. $0.777\dots$
e. $2 + \sqrt{5}$ f. $2\sqrt{12}$

Do the following if you need more practice.

2. Why is $\sqrt{\frac{4}{9}}$ a rational number?
3. What is an irrational number?



For solutions to **Activity 1**, turn to the **Appendix, Topic 2**.

Remember: Numbers that do not repeat or terminate are called **irrational numbers**.



Activity 2



Find approximations for square roots of non-perfect squares.

You are now aware that $\sqrt{2}$ is an irrational number. However, a sequence of digits that repeats could occur well into the decimal expansion. Awareness is not enough for a mathematician. A logical proof is needed.

You may want to look at the indirect proof that $\sqrt{2}$ is irrational in the **Extensions** section.

First the meaning of perfect squares will be clarified before you look at non-perfect squares.

A perfect square is a number that results from the multiplication of two identical rational numbers.

The number 9 is a perfect square since it can be factored as 3×3 . So $\sqrt{9}$ is equal to 3. Since 3 is a rational number, $\sqrt{9}$ is a rational number.

A non-perfect square is a number that does not have two identical rational factors.

The number 6 is not a perfect square. Its factors are 1×6 or 2×3 . Neither 1 and 6 nor 2 and 3 are identical factors. Therefore, $\sqrt{6}$ is an irrational number ($\sqrt{6} \doteq 2.449\,4897\dots$).

Study the following example and you will see the difference.

Example 2

Identify each number as either a perfect square or a non-perfect square, and state whether it is rational or irrational.

• $\sqrt{5}$

Solution:

The number 5 is a non-perfect square. It does not have identical factors. Thus, $\sqrt{5}$ is irrational.

On a ten-digit display calculator, $\sqrt{2} = 1.414\,213\,562\dots$ A calculator with a very large display will show more decimal digits and perhaps these may contain repeating decimals.

When a number has the same factors, the factors are identical to each other.

$$\bullet \sqrt{\frac{9}{16}}$$

Solution:

The fraction $\frac{9}{16}$ is a perfect square.

$$\frac{9}{16} = \frac{3 \times 3}{4 \times 4}$$

$$\sqrt{\frac{9}{16}} = \frac{3}{4}$$

Thus, $\sqrt{\frac{9}{16}}$ is rational.

$$\bullet \sqrt{\frac{4}{7}}$$

Solution:

The fraction $\frac{4}{7}$ is a non-perfect square. It does not have identical factors. Thus, $\sqrt{\frac{4}{7}}$ is irrational.



You can find the approximate square roots of irrational numbers (non-perfect squares) either by using a calculator or geometric constructions. Of course the calculator is faster. However, with the construction method you will be able to see exactly how the result is obtained.

Solution:

The number 36 is a perfect square.

$$36 = 6 \times 6$$

$$\sqrt{36} = 6$$

Thus, $\sqrt{36}$ is rational.

$$\bullet \sqrt{1.96}$$

Solution:

The decimal number 1.96 is a perfect square.

$$1.96 = 1.4 \times 1.4$$

Thus, $\sqrt{1.96}$ is rational.

The factors for $\frac{9}{16}$ are obtained by finding the two identical factors for the numerator and the two identical factors for the denominator.



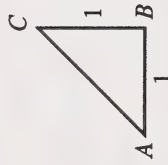
Study the following example.

Example 3

Find the square root of the number 2 by construction.

Solution:

The diagram shown here represents a right isosceles triangle with each equal side measuring one unit. Trace the ruler from the margin and find the length of the hypotenuse.



You can also find this length by calculation.

The length of the **hypotenuse** is the square root of the sum of the other two sides squared. See the **Extensions** section for a detailed study of the Pythagorean theorem.

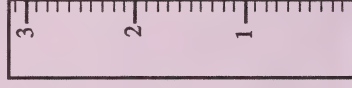
$$\begin{aligned}\text{Length of hypotenuse} &= \sqrt{1^2 + 1^2} \\ &= \sqrt{1+1} \\ &= \sqrt{2}\end{aligned}$$

Using a calculator, convert $\sqrt{2}$ to a decimal. This is approximately 1.4. How does this compare with your measured answer? They should be very close to each other.

You can find the square roots of other irrational numbers by similar constructions and measurements.



Use this ruler to find the measure of the hypotenuse for the right triangle shown.



The hypotenuse is the longest side of a right triangle.

Recall: The Pythagorean theorem states that $c^2 = a^2 + b^2$, where c represents the hypotenuse.

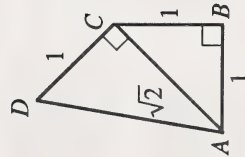
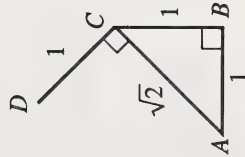
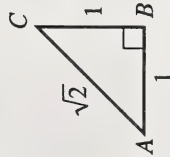
An isosceles triangle has two sides of equal measure.

Look at this example.

Example 4

Find the square root of 3.

Solution:



The measured value is approximately 1.7 units.

By calculation, the measure of the new hypotenuse $\overline{AD}$ is as follows:

$$\begin{aligned}\sqrt{1^2 + (\sqrt{2})^2} &= \sqrt{1+2} \\ &= \sqrt{3} \\ &\doteq 1.7\end{aligned}$$

The calculated value closely matches your measured value.

Try the following question.

Using the diagram in Example 4 and by further constructions and measurements, try to find the values of $\sqrt{4}$ and $\sqrt{5}$.



For solutions to Activity 2, turn to the Appendix, Topic 2.

If the angle between two lines is 90° , the two lines are perpendicular to each other. The symbol $\perp$ is used to show perpendicular lines.



Activity 3



Evaluate square roots and write the square roots of negative integers as imaginary numbers.

As you saw, $\sqrt{n}$ refers to a number that when squared results in a product of n . That is, if $\sqrt{n} = a$, then $a^2 = a \times a = n$.

Look at the following examples.

$$\sqrt{4} = 2 \text{ because } 2^2 = 2 \times 2 = 4$$

$$\sqrt{8} \doteq 2.828\,4271 \text{ because } (2.828\,4271)^2 \doteq 8$$

Remember that $\sqrt{8}$ is an irrational number because it is a non-repeating decimal.

$$\sqrt{9} = 3 \text{ because } 3^2 = 3 \times 3 = 9$$

But -3×-3 or $(-3)^2$ is also equal to 9. Therefore, -3 is also the square root of 9. When you write square root of 9 as $\sqrt{9}$, it means the **principal** square root of 9. That is $\sqrt{9} = 3$.



The principal square root of a number is the positive square root of the number.

A simple rule to remember is that $\sqrt{n}$ denotes the principal square root of a number.

Try the following.

1. $\sqrt{25} = \underline{\hspace{1cm}}$
because $\underline{\hspace{1cm}}^2 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = 25$
2. $\sqrt{16} = \underline{\hspace{1cm}}$
because $\underline{\hspace{1cm}}^2 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
3. $\sqrt{\frac{9}{25}} = \underline{\hspace{1cm}}$
because $\left(\frac{3}{5}\right)^2 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$



For solutions to **Activity 3**, turn to the **Appendix, Topic 2**.

By using a calculator you can find decimal representations for square roots. You can also **round** to a specified place value.



Example 5

Find $\sqrt{34}$ to three decimal places.

Solution:

When you press 34 followed by $\sqrt{\hspace{0.5cm}}$ on a calculator with a ten-digit display, the display shows 5.830951895.

Therefore, to three decimal places, the square root of 34 is about 5.831.

Example 6

- Evaluate $\sqrt{5.79}$ to the nearest tenth.

Solution:

$$\sqrt{5.79} = 2.4062418\dots$$

To the nearest tenth, $\sqrt{5.79}$ rounds to 2.4.

- Evaluate $\sqrt{5.79}$ to the nearest hundredth.

Solution:

$$\sqrt{5.79} = 2.4062418\dots$$

To the nearest hundredth, $\sqrt{5.79}$ rounds to 2.41.

Recall: The rule for rounding numbers are covered in the **Review** section.

You saw earlier that numbers can be rational or irrational. If an exact square root of a positive number can be found, such a number is rational. If only an approximate square root can be found (i.e. $\sqrt{2} = 1.414\ 2135\ldots$), such a number is irrational.

What do you suppose happens when you try to find the square root of a negative number? For example, what is $\sqrt{-4}$?



Try this on your calculator.
What happened? (It was not irrational; it was just an error.)

The result is neither rational nor irrational. You cannot square any number and get a negative result.

Recall that $(2)^2 = 4$ and $(-2)^2 = 4$.

When you square a negative number, the answer is a positive number.

What do you do with $\sqrt{-4}$? Do you see that if you can remove the negative from the radicand -4 , then you can find the square root of 4? To do this you will use a new mathematical symbol i that has the special property such that when you square it, you obtain -1 .

$$i^2 = -1$$

$$\sqrt{i^2} = \sqrt{i \times i} = i$$

$$i = \sqrt{-1}$$

Therefore, $\sqrt{-4}$ becomes $\sqrt{(4)(-1)}$.

$$\sqrt{(4)(-1)} = \sqrt{4} \times \sqrt{-1}$$

$$= 2 \times i$$

$$= 2i$$

Look at a negative fraction written as a radical.

$$\sqrt{-\frac{16}{9}} = \frac{4}{3}i$$

You try this one:

$$\sqrt{-24} = \sqrt{(\quad)(-1)}$$

$$= \sqrt{\quad} \times \sqrt{-1}$$

$$= \quad i$$

The answers for the previous two questions are located in the margin box.

Error means that the question asked has no solution, not that it is wrong in any way.

$$\begin{aligned}\sqrt{-\frac{16}{9}} &= \sqrt{\frac{16}{9} \times -1} \\ &= \sqrt{\frac{16}{9}} \times \sqrt{i^2} \\ &= \frac{4}{3}i\end{aligned}$$

$$\begin{aligned}\sqrt{24} &= \sqrt{(24)(-1)} \\ &= \sqrt{24} \times \sqrt{-1} \\ &= 4.90i\end{aligned}$$

This pattern can be extended to the square root of any negative number. You can conclude that the square roots of negative numbers belong to a number system using these i symbols. The i symbols are part of the imaginary number which are part of the number system called **complex numbers**.

Here are some examples.

Example 7

- Simplify $\sqrt{-9}$.

Solution:

$$\begin{aligned}\sqrt{-9} &= \sqrt{9 \times (-1)} \\ &= \sqrt{9} \times \sqrt{-1} \\ &= \sqrt{9} \times \sqrt{i^2} \\ &= 3i\end{aligned}$$

- Simplify $\sqrt{-98}$.

Solution:

$$\begin{aligned}\sqrt{-98} &= \sqrt{98 \times (-1)} \\ &= \sqrt{98} \times \sqrt{-1} \\ &= \sqrt{98} \times \sqrt{i^2} \\ &= \sqrt{98}i \\ &\doteq 9.899i\end{aligned}$$

- Simplify $\sqrt{-108}$.

Solution:

$$\begin{aligned}\sqrt{-108} &= \sqrt{108 \times (-1)} \\ &= \sqrt{108} \times \sqrt{-1} \\ &= \sqrt{108} \times \sqrt{i^2} \\ &= \sqrt{108}i \\ &\doteq 10.392i\end{aligned}$$

- Simplify $\sqrt{-144}$.

Solution:

$$\begin{aligned}\sqrt{-144} &= \sqrt{144 \times (-1)} \\ &= \sqrt{144} \times \sqrt{-1} \\ &= \sqrt{144} \times \sqrt{i^2} \\ &= 12i\end{aligned}$$

Now try these questions on your own.



- Evaluate each to the nearest hundredth.

- $\sqrt{27}$
- $\sqrt{60}$
- $-\sqrt{162}$
- $\sqrt{2.345}$
- $-\sqrt{\frac{16}{49}}$

- Simplify.

- $\sqrt{-64}$
- $\sqrt{-75}$ (to the nearest hundredth)
- $\sqrt{-81}$
- $\sqrt{-2000}$ (to the nearest hundredth)

- The area of a square is 225 cm^2 . Find the length of one side.

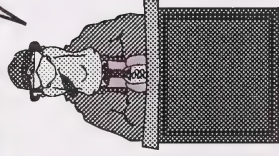
- Explain why $\sqrt{25}$ is a positive number.

- If $A = 169 \text{ cm}^2$ represents the area of a square, and the square root of 169 is 13 or -13 , explain why -13 is not acceptable.



For solutions to **Activity 3**, turn to the **Appendix, Topic 2**.

Hint: Remember i has a special property. When you square i , you obtain -1 .



Activity 4



Recognize π as a special irrational number.

You should be aware that the formula to find the circumference of a circle is $C = \pi d$. When this expression is written as $\pi = \frac{C}{d}$, it suggests that π is rational.

It has been proven by mathematicians that π cannot be expressed as a fraction in the form $\frac{m}{n}$, where m and n are integers and $n \neq 0$. Here is a brief history of π .

In 470 A.D. the Chinese used $\frac{355}{113}$ for π .

In 530 A.D. the Hindu value for π was set at $\frac{3927}{1250}$.

In 1220 A.D. a European estimate of π was given as $\frac{864}{275}$.

In 1761, Johann Lambert proved that π is irrational.

In 1986, π was computed to 29 360 128 decimal places with no repetition.

This fascinating number will be used throughout your study of mathematics. It occurs in many mathematical formulas including the volume of a cone $(V = \frac{1}{3} \pi r^2 h)$ and the volume of a sphere $(V = \frac{4}{3} \pi r^3)$.

If you want to expand your knowledge of rational and irrational numbers, study the **Extensions** section.

1. Compare the various values for π , given in this activity, with the present value of π (3.141 592 654). Did the values improve over time?
2. Find the circumference of a circle with a diameter of 12 cm. Use the value of π from your calculator. Round your answer to the nearest hundredth.



For solutions to **Activity 4**, turn to the **Appendix, Topic 2**.



If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

} You may decide to do both.



Extra Help

An irrational number can neither be represented by a fraction, a terminating decimal, nor a repeating decimal. It may be represented by a non-repeating decimal. Some examples of irrational numbers are as follows:

$$\sqrt{2}$$

$$\sqrt{5}$$

$$10.003\,332\,222\,839\,094\,64\ldots$$

$$\pi$$

If you want to change an irrational number to a decimal, you have to find its rational approximation.

For example, $\pi \div 3.1416$ to four decimal places and $\sqrt{2} \div 1.414$ to three decimal places. When rounding, discard the unnecessary digits. Add 1 to the last retained digit if the next digit is 5 or greater.

The square root of a negative number is not an irrational number. It is called a complex number. The set of complex numbers is a much larger set of numbers. The set of irrational numbers is a part of the set of complex numbers. An irrational number is a special kind of complex number.

If you want to know more about the relationships between the number systems, study **Topic 3** carefully.

The symbol i is used to denote $\sqrt{-1}$. Therefore, the complex number $\sqrt{-3}$ can be changed to $\sqrt{3}i$ or $i\sqrt{3}$.

$$\begin{aligned}\sqrt{-3} &= \sqrt{3}\sqrt{-1} \\ &= \sqrt{3} \times \sqrt{-1} \\ &= \sqrt{3} \times \sqrt{i^2} \\ &= i\sqrt{3}\end{aligned}$$

In order to determine whether the square root of a number is an irrational number, determine whether the number is a perfect square. If the number is a perfect square (such as 0, 4, 9, 16, or 25), then its square root is rational. If the number is not a perfect square, then its square root is irrational.

For example, consider $\sqrt{18}$. Since 18 is not a perfect square, $\sqrt{18}$ is an irrational number.

Now try some practice questions.

1. Identify each of the following numbers as a rational number or an irrational number.

a. $\sqrt{7}$

b. 5.231451687 ...

c. $\frac{2}{3}$

d. $5.\overline{23}$

e. 8.712

2. Express each in one of the following forms: $\sqrt{a}$, $-\sqrt{a}$, $\sqrt{ai}$, or $-\sqrt{ai}$.

a. $\sqrt{-7}$

b. $-\sqrt{-8}$

c. $-\sqrt{27}$

d. $\sqrt{125}$



For solutions to **Extra Help**, turn to the **Appendix, Topic 2**.

Hint: $i = \sqrt{-1}$
 $i^2 = -1$



Extensions

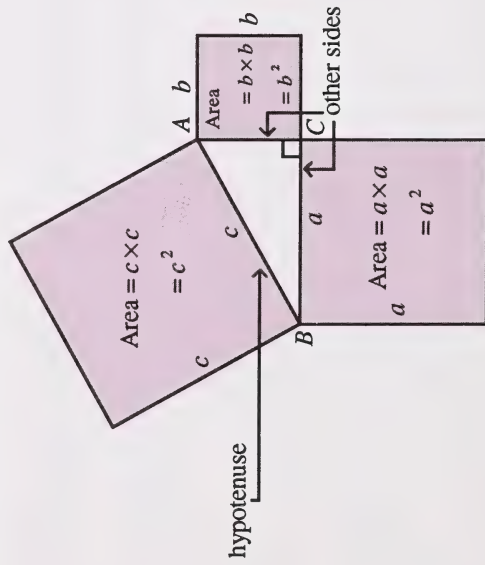
The Pythagorean Theorem

Pythagoras, a Greek mathematician, discovered this theorem in the 6th century B.C. The theorem states the following:

In any right triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.

To understand what this means, look at the triangle in the diagram. Triangle ABC is a right triangle where c is the hypotenuse. By using the formula $c^2 = a^2 + b^2$, you can find the unknown side of this triangle if the other two sides are given.

You can also write this formula as $(AB)^2 = (BC)^2 + (AC)^2$.



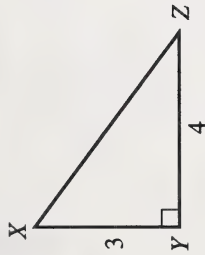
A right triangle is a triangle that contains a right angle.

The expression $c^2 = a^2 + b^2$ may be written as $a^2 = c^2 - b^2$ or $b^2 = c^2 - a^2$.

Now look at some examples.

Example 8

Find the length of the hypotenuse in triangle XYZ.



Solution:

First, write the formula.

$$(XZ)^2 = (XY)^2 + (YZ)^2$$

Now substitute the values for XY and YZ and solve.

$$(XZ)^2 = (3)^2 + (4)^2$$

$$(XZ)^2 = 9 + 16$$

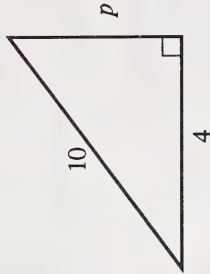
$$(XZ)^2 = 25$$

$$\sqrt{(XZ)^2} = \sqrt{25}$$

$$XZ = 5$$

Example 9

Find the value of p in the right triangle.



Solution:

First, write the defining equation.

$$10^2 = 4^2 + p^2$$

Isolate p^2 and solve for p .

$$p^2 = 10^2 - 4^2$$

$$p^2 = 100 - 16$$

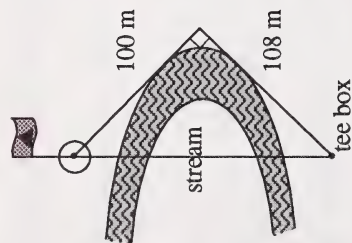
$$p^2 = 84$$

$$\sqrt{p^2} = \sqrt{84}$$

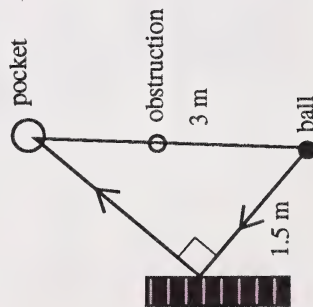
$$p \doteq 9.2 \quad (\text{to one decimal place})$$

Try using the Pythagorean theorem in the following questions.

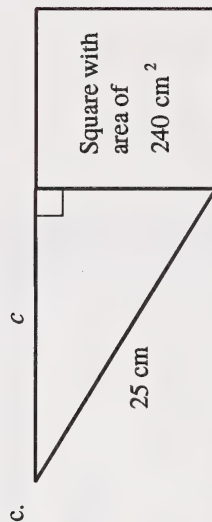
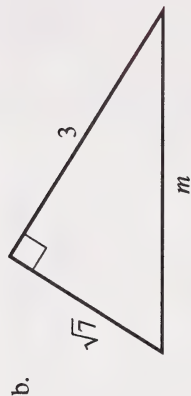
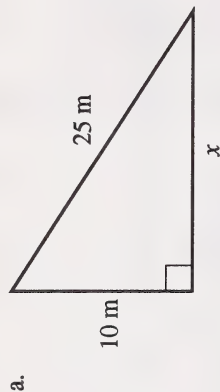
1. The diagram represents the distances along the fairway from the tee box to the flag. Calculate the distance from the tee box to the flagpost across the stream. Give your answer to the nearest metre.



2. In a game of billiards, due to an obstruction, a ball has to be shot at a right angle to the hole so that it will deflect off the side and go in. Calculate the distance the ball travels from the side of the billiard table to the pocket. Give your answer to one decimal place.



3. Find the unknown side. Round to one decimal place.



For solutions to Extensions, turn to the Appendix, Topic 2.

Proving that $\sqrt{2}$ Is Irrational

Let $\sqrt{2} = \frac{a}{b}$ where $a, b \in I$ and $b \neq 0$.

Note that $\frac{a}{b}$ is a rational number in lowest terms.

Squaring both sides you get the following:

$$\begin{aligned}(\sqrt{2})^2 &= \left(\frac{a}{b}\right)^2 \\ 2 &= \frac{a^2}{b^2}\end{aligned}$$

By cross multiplication, the result is as follows:

$$2b^2 = a^2$$

Since b^2 is multiplied by 2, $2b^2$ must be an even number. Therefore, a^2 is and so is a .

Now a can be written as $2n$ for $n \in I$.

$$\begin{aligned}\text{Therefore, } a^2 &= (2n)^2 \\ &= 4n^2 \\ \text{and } 4n^2 &= 2b^2\end{aligned}$$

This means that b^2 must be even; therefore, b must also be even. Thus, a and b have a common factor since they are even. This is a contradiction from the first assumption that said $\frac{a}{b}$ was in lowest terms.

Therefore, $\sqrt{2}$ cannot be written as $\frac{a}{b}$ and must be irrational.

Now do the following question.

4. Prove that $\sqrt{3}$ is an irrational number.



For solutions to Extensions, turn to the **Appendix, Topic 2**.



I = the set of Integers

$\{\dots -3, -2, -1, 0, 1, 2, 3, \dots\}$

Note that a and b cannot have a common factor since $\frac{a}{b}$ is a rational number in lowest terms.

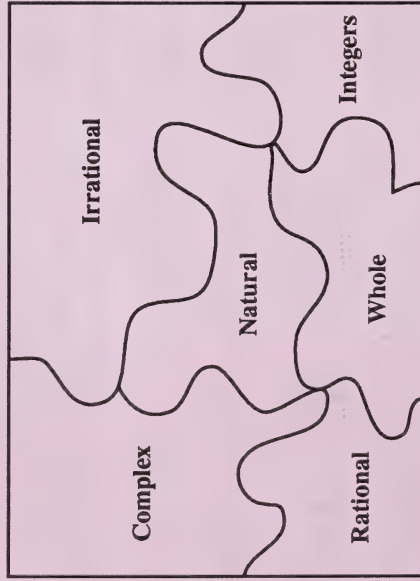
Topic 3 Relationships Among Number Systems



Introduction

A jigsaw puzzle has many pieces that fit together to form a whole picture. So far you have many number system pieces to fit into the big picture of number systems. Unlike jigsaw puzzle pieces, the pieces of your number systems puzzle can and do overlap. That is, some numbers belong to more than one system.

Number Systems



What Lies Ahead

Throughout this topic you will learn to

1. identify real numbers as the union of rational and irrational numbers
2. identify complex numbers as the union of real and imaginary numbers

Now that you know what to expect, turn the page to begin your study of relationships among number systems.



Exploring Topic 3

Activity 1

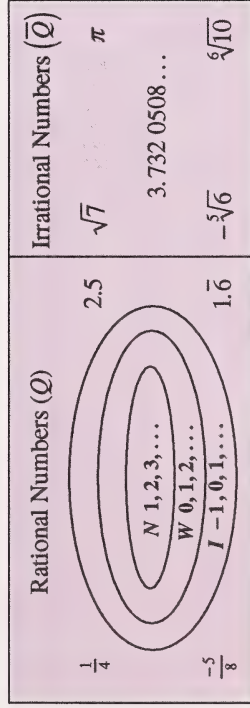


Identify real numbers as the union of rational and irrational numbers.

In this unit you learned that rational numbers can be written as decimals that repeat or terminate. You also learned that irrational numbers can be represented with decimals that neither terminate nor repeat. These two systems of numbers can be combined to form one system of numbers called the real number.

The following diagram illustrates this.

Real Numbers (R)



Each number system has special symbols. You must clearly understand the meaning of the symbols as they will occur frequently throughout your study of mathematics.

N = set of natural numbers
 W = set of whole numbers
 I = set of integers
 Q = set of rationals
 $\bar{Q}$ = set of irrationals
 R = set of real numbers

A set is a collection of articles that have some common characteristic. Two examples are a set of dishes or a set of baseball cards.

Numbers also belong to sets because of their common characteristics.

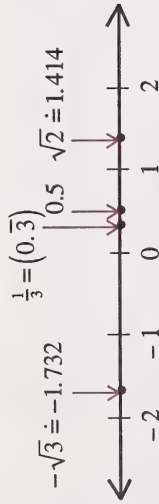
Rational and irrational numbers are two separate pieces that join together to give one picture: the real numbers.

$$R = Q \cup \bar{Q}$$

The symbol $\cup$ means **union**, and union means the formation of a new larger group by combining two or more smaller groups.

Locating Real Numbers on a Line

A line is composed of points. There are millions and millions, in fact, an infinite number, of points in a line. Each point corresponds to a real number. This is referred to as a 1:1 correspondence.



You could place more numbers on the real number line by converting all the numbers to decimals.

Try some of the following exercise.

Remember that irrational numbers are numbers that do not repeat or terminate.

- Classify each of the following as a rational number (Q) or an irrational number ($\overline{Q}$).

a. 12

b. π

c. $1.\overline{27}$

d. 4.15115115...

e. $-\sqrt{11}$

f. $\frac{1}{3}$

g. 5.287

h. $\sqrt{\frac{4}{18}}$

- Locate each of the following on the number line.

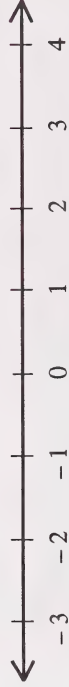
a. 2

b. $\frac{7}{4}$

c. -0.2

d. $\sqrt{6}$

e. $0.\overline{7}$



For solutions to Activity 1, turn to the **Appendix, Topic 3**.



Activity 2



Identify complex numbers as the union of real and imaginary numbers.

In an earlier section of this unit, you learned how to take the square root of a negative number. This required the use of a new symbol i .

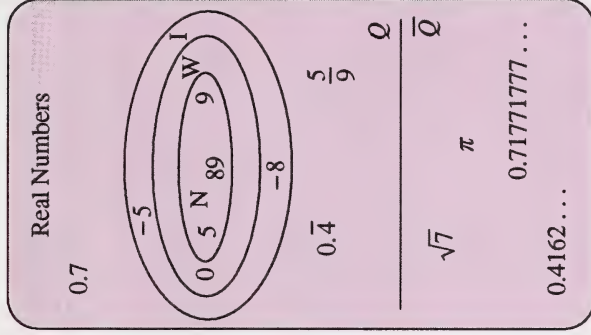
Some authors suggest that the letter i can be considered to represent imaginary numbers. This symbol becomes the basis for a new set of numbers called complex numbers. Complex numbers are the union of real numbers and imaginary numbers.

The real numbers are part of (or included within) the complex numbers.

The real numbers and the imaginary numbers combine to form the complex numbers.

The following attempts to give a visual representation of the two relationships.

Complex Numbers



Therefore, complex numbers can be viewed as the union (or combination) of real numbers and imaginary numbers.

Try the following question.

Describe the complex number system.



For solutions to **Activity 2**, turn to the **Appendix, Topic 3**.

If you require help, do the Extra Help section.

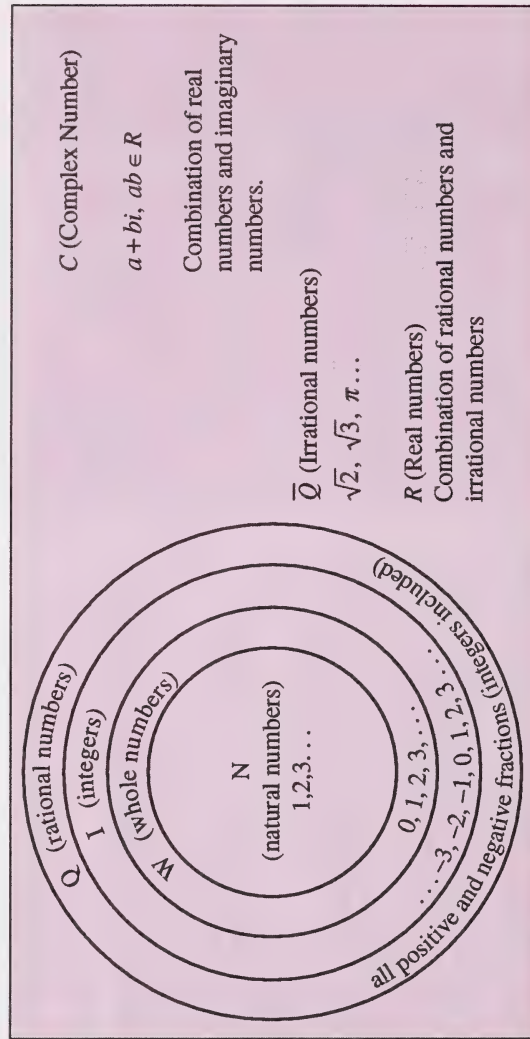
If you want more challenging explorations, do the Extensions section.

} You may decide to do both.



Extra Help

The following diagram is called the Venn diagram. It illustrates the relationships among all the number systems you have learned.



How can a real number be a complex number? Real numbers are all numbers that are not imaginary. They do not include $\sqrt{-1}$ or i . The i in the expression $a + bi$ is cancelled when $b = 0$. Therefore, a real number is a complex number with $b = 0$. The number 24 is a real number. It can be written in complex number form as $24 + 0i$.

From the diagram, you can tell that the set of complex numbers is the largest set of numbers. The set of real numbers is only a part of or a subset of the set of complex numbers. In other words, real numbers are special complex numbers. The set of real numbers consists of rational and irrational number. In the set of rational numbers you can see that the set of natural numbers is the subset of whole numbers. The set of whole numbers is the subset of the set of integers. The set of integers is the subset of rational numbers.

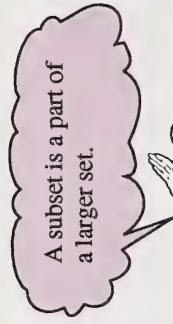
Now answer the questions that follow.

Use the relationship diagram to answer the following questions.

1. Is set W a subset of N ?
2. Is set I a subset of R ?
3. Is set Q a subset of W ?
4. Is set $\bar{Q}$ a part of a set C ?
5. Is set $\bar{Q}$ a subset of Q ?
6. Is the set R a part of a set C ?
7. Is an integer a real number?
8. Is a whole number a complex number?



For solutions to **Extra Help**, turn to the **Appendix, Topic 3**.



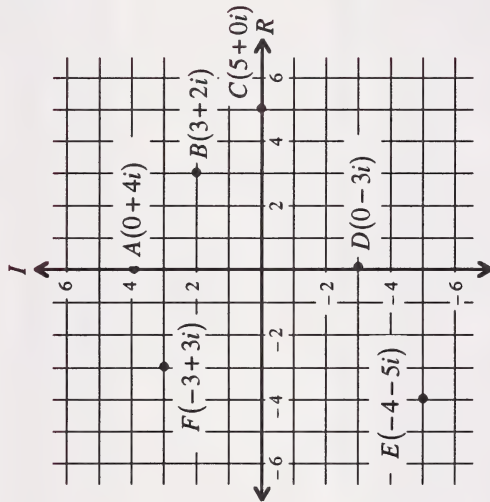


Extensions

The geometric representation of the set of real numbers is the number line. For every real number, there is only one point on the number line that corresponds to that real number. Conversely, for every point on the number line, there is only one corresponding number in the set of real numbers. In other words, the number line is filled up by real numbers. Where can the imaginary numbers be placed?

The standard form of a complex number is $a + bi$, where a and b are real numbers. It follows that an ordered pair (a, b) can be used to represent a complex number. The geometric representation of a complex number $a + bi$ is the point $P(a, b)$ of the coordinate plane. The plane is usually called the complex plane. The horizontal axis is called the axis of reals and the vertical axis is called the axis of imaginaries.

Examples are shown here.



In the previous diagram, the points A , B , C , D , E , and F correspond to the following complex numbers.

$$A(0, 4) \rightarrow 4i$$

$$B(3, 2) \rightarrow 3 + 2i$$

$$C(5, 0) \rightarrow 5 + 0i = 5 \text{ (real number)}$$

$$D(0, -3) \rightarrow 0 - 3i = -3i$$

$$E(-4, -5) \rightarrow -4 - 5i$$

$$F(-3, 3) \rightarrow -3 + 3i$$

Examples of complex numbers include the following:

$$0 + \sqrt{2}i$$

$$2 + \sqrt{3}i$$

$$5 + 4i$$

$$7i$$

Note the following:

$$0 + \sqrt{2}i = \sqrt{2}i$$

$$0 + 7i = 7i$$

1. Write the ordered pairs that correspond to the following complex numbers. Plot the points in the coordinate plane.

a. $3 - 2i$

b. $(5 + 7i)$

c. $8i$

d. 6

2. Write the complex numbers represented by the following ordered pairs.

a. $(3, 7)$

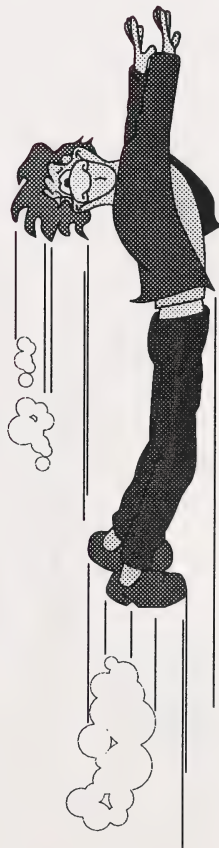
b. $(0, 5)$

c. $(9, 0)$

d. $(-2, -7)$



For solutions to Extensions, turn to the Appendix,
Topic 3.



Recall: The standard form of a complex number is $a + bi$, where a and b are real numbers.

Unit Summary



What You Have Learned

You have studied the following topics:

- operations with rational numbers
- changing the various forms of rational numbers
- examining irrational numbers
- identifying real numbers
- identifying the origin of the complex numbers and their relationship with the real numbers

You are now ready to
complete the **Unit Assignment**.

Appendix



Solutions

Review

Topic 1 Rational Number System

Topic 2 Irrational Number System

Topic 3 Relationships Among Number Systems



Review

1. a. $-9 + 82 = 73$

b. $6 + (-21) = -15$

c. $-5 + (-7) = -12$

2. a. $-4 - 9 = -13$

b. $5 - 2 = 3$

c. $-7 - (-12) = -7 + 12$
 $= 5$

3. a. $3(-5) = -15$

b. $-4(-8) = 32$

c. $-15(5) + (-6)(2) = (-75) + (-12)$
 $= -87$

4. a. $72 \div 9 = 8$

b. $-63 \div 21 = -3$

c. $-125 \div (-5) = 25$

5. a. $\frac{7(-12) - 7(22)}{3(5)(-7) + 7(13)} = \frac{-84 - 154}{-105 + 91}$
 $= \frac{-238}{-14}$
 $= 17$

b. $15 \div (4 + 3 - 2) \times (8 - 11 + 2) = 15 \div (5) \times (-1)$
 $= 15 \div (5) \times (-1)$
 $= 15 \div (-5)$
 $= -3$

6. a. $5.002 + 0.07 + 18.6 = 23.672$

b. $4.893 - 3.02 = 1.873$

c. $0.5217 \times 0.006 = 0.0031302$

d. $756.82 \div 0.016 = 47301.25$

7. a. 16.825

To the nearest one, it is 17.

The 6 changes to 7 since the digit to the right of 6 is greater than 4.

To the nearest ten, it is 20.

The 1 changes to 2 since the digit to the right of 1 is greater than 4.

To the nearest tenth, it is 16.8.

The 8 does not change since the digit to the right of it is less than 5.

To the nearest hundredth, it is 16.83.

The 2 changes to 3 since the digit to the right of 2 is greater than 4.

b. 0.462

To the nearest one, it is 0.

The 0 does not change since the digit to the right of 0 is less than 5.

To the nearest ten, it is 0.

The 0 does not change since the digit to the right is less than 5.

To the nearest tenth, it is 0.5.

The 4 changes to 5 since the digit to the right of 4 is greater than 4.

To the nearest hundredth, it is 0.46.

The digit 6 does not change since the digit to the right of it is less than 5.

8. a. $\frac{15}{45} = \frac{1}{3}$

b. $\frac{7}{49} = \frac{1}{7}$

$\frac{5}{15} = \frac{1}{3}$

$\frac{3}{21} = \frac{1}{7}$

c. $\frac{36}{81} = \frac{4}{9}$

$\frac{8}{18} = \frac{4}{9}$

9. a. $5 \times 7 = 35$

b. $4 \times 11 \times 3 = 132$

10. First find the lowest common denominator and write equivalent fractions.

$\frac{1}{2} = \frac{15}{30}$

$\frac{2}{5} = \frac{12}{30}$

$\frac{1}{6} = \frac{5}{30}$

$\frac{1}{3} = \frac{10}{30}$

In ascending order the fractions are $\frac{1}{6}$, $\frac{1}{3}$, $\frac{2}{5}$, and $\frac{1}{2}$.

11. a. $5\frac{1}{2} + 2\frac{1}{3} = 5\frac{3}{6} + 2\frac{2}{6}$

$= 7\frac{5}{6}$

b. $3 - 1\frac{4}{7} = 2\frac{7}{7} - 1\frac{4}{7}$

$= 1\frac{3}{7}$

c. $3\frac{1}{3} \times 1\frac{2}{7} = \frac{10}{3} \times \frac{9}{7}$

$= \frac{30}{7}$

$= 4\frac{2}{7}$

d. $6\frac{1}{2} + \frac{13}{14} = \frac{13}{2} + \frac{14}{14}$

$= 7$

2. $\frac{1}{7} = 0.142857$ $\frac{3}{7} = 0.428571$ $\frac{6}{7} = 0.857142$

The repeating block of digits is one less than the denominator. (Six digits repeat and the denominator is 7.)

3. a. Goals against average is equal to the number of scored divided by the number of games played.

b. A hockey games takes sixty minutes to play. The goalkeeper played for twenty minutes. Therefore, the goalkeeper played for $\frac{20}{60} = \frac{1}{3}$ of a game.

goals against average = number of goals + games played

$= 2 + \frac{1}{3}$

$= 2 \times \frac{3}{1}$

$= 6$

The goalkeeper's goals against average is 6 goals per game.

4. a.

Players	Number of Successful Smashes	Number of Tries	Ratio successful smashes number of tries
Kim Choi	5	8	$\frac{5}{8} = 0.625$
Darcy Lowe	8	11	$\frac{8}{11} = 0.\overline{72}$



Exploring Topic 1

Activity 1

Identify repeating or terminating decimals as rational numbers.

1. A repeating decimal is a decimal that repeats infinitely. A terminating decimal is a decimal expressed as a finite number of figures or digits.

b. Darcy is the better smasher.

5. The repeating decimals are b, c, and e.

Activity 2

Change fractions to decimals and percents.

1. A fraction is the most appropriate comparison for this calculation. The exact value is found by changing two months to part of a year and computing.

$$\frac{2}{12} \times \$36\,000 = \$6000$$

Joseph's loss in pay was \$6000.

The decimal form repeats and this will give an approximate loss in pay.

$$\frac{2}{12} = 0.\overline{16}$$

The percent will also give an approximation.

$$\frac{2}{12} = n$$

$$n = 16.\overline{6}\% \div 17\%$$

2. The consumer is familiar with the use of percents for sale items. Mental calculations are made easily and quickly with percents but not with decimals. It is also more work for the business person in marking up articles using decimals for comparing profits and losses. Decimals can be confused with the price in dollars and cents.

3. $\frac{9}{11} = 0.\overline{18} \div 82\%$

It is more meaningful to the audience to use fractions or percents. You can make an easy comparison. You can tell very quickly if the quarterback is performing well. The decimal does not register that well especially if it is repeating. You would want to convert back to a percent or a fraction so that it would make more sense to you.

4. This is the most practical way to compare the parties. The poll is a random sampling. The results are best reflected as a percent. It is easy to compare the parties' standings.

5. a. decimal
b. percent
c. fraction or decimal
d. percent
e. percent
f. percent
g. fraction

6. You could do this activity by making squares on a sheet of paper, cutting away the corners, and folding up the sides.

Total number of squares is $5 \times 7 = 35$

All the outside squares form the cut-away and sides that are folded. In total they represent 20 squares. The bottom has 15 squares.

$$\frac{15}{35} = \frac{3}{7}$$

For the bottom of the tray, $\frac{3}{7}$ of the paper was used.

$$\frac{3}{7} \div 0.429 \\ \doteq 42.9\%$$

As a percent, $\frac{3}{7}$ is equal to 42.9%.

7.

Fraction	Decimal	Percent
$\frac{7}{8}$	0.875	87.5%
$\frac{151}{200}$	0.755	75.5%
$\frac{7}{10}$	0.7	70%

8.

Fraction	Decimal	Percent
$\frac{2}{4} = \frac{1}{2}$	0.50	50%
$\frac{4}{9}$	$0.\bar{4}$	$44.\bar{4}\%$
$\frac{1}{4}$	0.25	25%

- a.
b.
c.

Activity 3

Change terminating decimals to fractions and percents.

1. Fractions would best describe a part of the orange. The comparison could be easily seen.

$$2. \quad a. \quad 0.57 = \frac{57}{100} = \frac{57}{100} = 57\%$$

$$b. \quad 16.4 = 16\frac{4}{10} \\ = 16\frac{2}{5} \\ = 16\frac{40}{100} \\ = 1640\%$$

$$\begin{aligned} \text{c. } 128.426 &= 128 \frac{426}{1000} \\ &= 128 \frac{213}{500} \end{aligned}$$

$$\begin{aligned} 128 \frac{426}{1000} &= \frac{128\,426}{1000} \\ &= \frac{12\,842.6}{100} \\ &= 12\,842.6\% \end{aligned}$$

Activity 4

Recognize fractional equivalents of common repeating decimals (thirds, sixths, ninths).

$$1. \quad \frac{2}{3} = \underline{0.\overline{6}}$$

$$2. \quad \frac{4}{3} = \underline{1.\overline{3}}$$

$$3. \quad \frac{1}{9} = \underline{0.\overline{1}}$$

$$4. \quad \frac{2}{9} = \underline{0.\overline{2}}$$

$$5. \quad \frac{3}{9} = \underline{0.\overline{3}}$$

$$6. \quad \frac{2}{6} = \underline{0.\overline{3}}$$

$$7. \quad \frac{3}{6} = \underline{0.5}$$

$$8. \quad \frac{4}{6} = \underline{0.\overline{6}}$$

Extra Help

$$1. \quad \text{a. } -1$$

$$\text{b. } -15$$

$$\text{c. } -8$$

$$\text{d. } 15$$

$$\text{e. } 10$$

$$\text{f. } -35$$

$$2. \quad \text{a. } 7$$

$$\text{b. } -14$$

$$\text{c. } 6$$

$$\text{d. } 15$$

$$\text{e. } 22$$

$$\text{f. } 13$$

$$3. \quad \text{a. } -66$$

$$\text{b. } 36$$

$$\text{c. } -100$$

$$\text{d. } 35$$

$$\text{e. } 90$$

$$\text{f. } 288$$

$$4. \quad \text{a. } 5$$

$$\text{b. } 5$$

$$\text{c. } -3$$

$$\text{d. } -8$$

$$5. \quad \text{a. } -28$$

$$\text{b. } 17$$

$$6. \quad \text{a. } 3+4=7$$

$$\text{b. } 6 \div 3 = 2$$

$$\begin{aligned} \text{c. } \frac{15+3}{4 \times 2 - 7} &= \frac{5}{8-7} \\ &= \frac{5}{1} \\ &= 5 \end{aligned}$$

$$\begin{aligned} \text{d. } 15 + (5) \times (-1) &= 3 \times (-1) \\ &= -3 \end{aligned}$$

$$\begin{aligned} \text{e. } \frac{16+4 \times 2}{19-17} &= \frac{4 \times 2}{2} \\ &= \frac{8}{2} \\ &= 4 \end{aligned}$$

$$\begin{aligned} \text{f. } \frac{[5 \times 2] + 2}{[4 + 2] + 4} &= \frac{10+2}{2+4} \\ &= \frac{12}{6} \\ &= 2 \end{aligned}$$

$$\begin{array}{r} 7. \quad 3.002 \\ 45.730 \\ + 12.800 \\ \hline 61.532 \end{array}$$

$$\begin{array}{r} 8. \quad 8.26 \\ - 4.30 \\ \hline 3.96 \end{array}$$

$$\begin{array}{r} 9. \quad 15.00 \\ - 9.37 \\ \hline 5.63 \end{array}$$

$$\begin{array}{r} 10. \quad 32.8 \\ \times 6.7 \\ \hline 2296 \\ 19680 \\ \hline 219.76 \end{array}$$

$$\begin{array}{r} 11. \quad 0.721 \\ \times 0.05 \\ \hline 0.03605 \end{array}$$

$$\begin{array}{r} 12. \quad 100.8 + 0.72 \\ \overset{140}{72} \overline{) 10080} \\ \underline{72} \\ 288 \\ \underline{288} \\ 00 \end{array}$$

$$\begin{array}{r} 13. \quad 0.9 = 0.3 \\ 3 \overline{) 0.9} \\ \underline{9} \\ 0 \end{array} \qquad \begin{array}{r} 3.4 \\ + 0.3 \\ \hline 3.7 \end{array}$$

$$14. \quad 5.5 + 11$$

$$\begin{array}{r} 0.5 \\ 11 \overline{) 5.5} \\ \underline{5.5} \\ 0 \end{array}$$

$$38.5 \div 0.5$$

$$\begin{array}{r} 77 \\ 5 \overline{) 385} \wedge \\ \underline{35} \\ 35 \\ \underline{35} \\ 0 \end{array}$$

15.

Number	Nearest			
	ten	one	tenth	hundredth
13.465	10	13	13.5	13.47
2.9745	0	3	3.0	2.97
8.905	10	9	8.9	8.91
384.782	380	385	384.8	384.78
7.398	10	7	7.4	7.40
0.902	0	1	0.9	0.90

16. a. $8 \times 15 = 10 \times 12$

$$120 = 120$$

$$\therefore \frac{8}{12} = \frac{10}{15}$$

b. $4 \times 15 = 3 \times 20$

$$60 = 60$$

$$\therefore \frac{4}{20} = \frac{3}{15}$$

- c. $18 \times 16 = 12 \times 24$
 $288 = 288$
 $\therefore \frac{18}{24} = \frac{12}{16}$
- d. $6 \times 40 = 16 \times 15$
 $240 = 240$
 $\therefore \frac{6}{15} = \frac{16}{40}$
- e. $4 \times 21 = 6 \times 14$
 $84 = 84$
 $\therefore \frac{4}{14} = \frac{6}{21}$
- f. $5 \times 16 = 4 \times 20$
 $80 = 80$
 $\therefore \frac{5}{20} = \frac{4}{16}$
17. a. $\frac{6}{10}, \frac{9}{15}, \frac{12}{20}, \frac{15}{25}, \frac{18}{30}, \dots$ (There are others.)
- b. $\frac{14}{20}, \frac{21}{30}, \frac{28}{40}, \frac{35}{50}, \frac{42}{60}, \dots$ (There are others.)
- c. $\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \frac{16}{20}, \frac{24}{30}, \dots$ (There are others.)
18. a. $2 = 2, 4, 6, 8, 10, \dots$
 $3 = 3, 6, 9, 12, \dots$
 $6 = 6, 12, 18, \dots$
Common multiples are 6, 12, 18, ...
L.C.M. = 6
- b. $3 = 3, 6, 9, 12, 15, 18, 21, 24, 27, \dots$
 $7 = 7, 14, 21, 28, 35, \dots$
Common multiples are 21, 42, 63, ...
L.C.M. = 21
- c. $12 = 12, 24, 36, 48, 60, \dots$
 $18 = 18, 36, 54, 72, \dots$
Common multiples are 36, 72, 108, ...
L.C.M. = 36
- d. $4 = 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, \dots$
 $60, 64, \dots$
 $5 = 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, \dots$
 $6 = 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, \dots$
Common multiples are 60, 120, 180, ...
L.C.M. = 60
19. a. $\frac{1}{3} + \frac{2}{5} = \frac{5}{15} + \frac{6}{15}$
 $= \frac{11}{15}$
- b. $\frac{3}{8} + \frac{3}{6} = \frac{9}{24} + \frac{12}{24}$
 $= \frac{21}{24}$
 $= \frac{7}{8}$
- c. $3\frac{1}{3} + 2\frac{1}{2} = \frac{10}{3} + \frac{5}{2}$
 $= \frac{20}{6} + \frac{15}{6}$
 $= \frac{35}{6}$ or $5\frac{5}{6}$

$$\begin{aligned} \text{d. } 2\frac{2}{3} + 5\frac{3}{4} &= \frac{8}{3} + \frac{23}{4} \\ &= \frac{32}{12} + \frac{69}{12} \\ &= \frac{101}{12} \text{ or } 8\frac{5}{12} \end{aligned}$$

$$\begin{aligned} \text{e. } \frac{1}{2} - \frac{1}{3} &= \frac{3}{6} - \frac{2}{6} \\ &= \frac{1}{6} \end{aligned}$$

$$\begin{aligned} \text{f. } \frac{3}{4} - \frac{2}{3} &= \frac{9}{12} - \frac{8}{12} \\ &= \frac{1}{12} \end{aligned}$$

$$\begin{aligned} \text{g. } 2 - 1\frac{5}{7} &= \frac{2}{1} - \frac{12}{7} \\ &= \frac{14}{7} - \frac{12}{7} \\ &= \frac{2}{7} \end{aligned}$$

$$\begin{aligned} \text{h. } 5\frac{1}{2} - 2\frac{3}{4} &= \frac{11}{2} - \frac{11}{4} \\ &= \frac{22}{4} - \frac{11}{4} \\ &= \frac{11}{4} \text{ or } 2\frac{3}{4} \end{aligned}$$

$$\text{i. } \frac{1}{5} \times \frac{7}{9} = \frac{7}{15}$$

$$\text{j. } \frac{2}{9} \times \frac{4}{5} = \frac{4}{15}$$

$$\begin{aligned} \text{k. } 2\frac{1}{2} \times \frac{5}{8} &= \frac{5}{2} \times \frac{5}{8} \\ &= \frac{25}{16} \text{ or } 1\frac{9}{16} \end{aligned}$$

$$\begin{aligned} \text{l. } 2\frac{1}{4} \times 1\frac{1}{5} &= \frac{9}{4} \times \frac{6}{5} \\ &= \frac{27}{10} \text{ or } 2\frac{7}{10} \end{aligned}$$

$$\begin{aligned} \text{m. } \frac{1}{2} + \frac{4}{3} &= \frac{1}{2} \times \frac{3}{3} \\ &= \frac{3}{8} \end{aligned}$$

$$\begin{aligned} \text{n. } \frac{4}{5} + \frac{8}{5} &= \frac{4}{5} \times \frac{1}{1} + \frac{8}{5} \times \frac{1}{1} \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{o. } 6\frac{1}{4} + 2\frac{1}{2} &= \frac{25}{4} + \frac{5}{2} \\ &= \frac{5}{2} \times \frac{1}{1} + \frac{5}{2} \times \frac{1}{1} \\ &= \frac{5}{2} \text{ or } 2\frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{p. } 5\frac{1}{2} + \frac{11}{12} &= \frac{11}{2} + \frac{11}{12} \\ &= \frac{1}{2} \times \frac{6}{6} + \frac{11}{12} \times \frac{1}{1} \\ &= \frac{6}{1} \text{ or } 6 \end{aligned}$$

Extensions

1. Convert each repeating decimal to a fraction. Find the product of the fractions, then convert it to a decimal.

$$\text{Let } x = 0.\dot{6}.$$

$$\begin{aligned} 10x &= 6.\dot{6} \\ -x &= 0.\dot{6} \\ \hline 9x &= 6.0 \quad (\text{by subtraction}) \end{aligned}$$

$$\begin{aligned} x &= \frac{6}{9} \\ &= \frac{2}{3} \end{aligned}$$

$$\text{Let } x = 0.8\dot{3}.$$

$$\begin{aligned} 10x &= 8.\dot{3} \\ -x &= 0.8\dot{3} \\ \hline 9x &= 7.50 \quad (\text{by subtraction}) \end{aligned}$$

$$\begin{aligned} x &= \frac{75}{90} \\ &= \frac{5}{6} \end{aligned}$$

Remember, for convenience of subtraction, to fill in the space with the repeating block.

$$\begin{aligned} \frac{2}{3} \times \frac{5}{6} &= \frac{10}{18} \\ &= \frac{5}{9} \\ &= 0.\dot{5} \end{aligned}$$

2. a. Let $x = 3.\overline{26}$.

$$\begin{array}{r} 100x = 326.\overline{26} \\ -x = 3.\overline{26} \\ \hline 99x = 323.00 \\ x = \frac{323}{99} \end{array}$$

b. Let $x = 0.02\overline{7}$.

$$\begin{array}{r} 10x = 0.2\overline{7} \\ -x = 0.02\overline{7} \\ \hline 9x = 0.250 \\ 900x = 25 \end{array}$$

$$x = \frac{25}{900} = \frac{1}{36}$$

c. Let $x = 11.23\overline{5}$.

$$\begin{array}{r} 1000x = 11235.\overline{235} \\ -x = 11.\overline{235} \\ \hline 999x = 11224.000 \\ x = \frac{11224}{999} \end{array}$$

3. a. $0.\overline{15} + 0.\overline{67} = 0.\overline{82}$

b. $0.\overline{73} - 0.\overline{51} = 0.\overline{22}$

4. a. Let $x = 0.\overline{2}$.

$$\begin{array}{r} 10x = 2.\overline{2} \\ -x = 0.\overline{2} \\ \hline 9x = 2 \\ x = \frac{2}{9} \end{array}$$

Let $x = 1.\overline{3}$

$$\begin{array}{r} 10x = 13.\overline{3} \\ -x = 1.\overline{3} \\ \hline 9x = 12 \\ x = \frac{12}{9} \\ = \frac{4}{3} \end{array}$$

Multiply the fractions.

$$\frac{2}{9} \times \frac{4}{3} = \frac{8}{27}$$

The decimal conversion is $0.\overline{296}$.

b. $0.75 = \frac{75}{100} = \frac{3}{4}$

Let $x = 2.\overline{6}$.

$$\begin{array}{r} 10x = 26.\overline{6} \\ -x = 2.\overline{6} \\ \hline 9x = 24 \\ x = \frac{24}{9} \\ = \frac{8}{3} \end{array}$$

The product of $\frac{3}{4}$ and $\frac{8}{3}$ is 2.



Exploring Topic 2

Activity 1

Recognizing irrational numbers as infinite, non-repeating decimals.

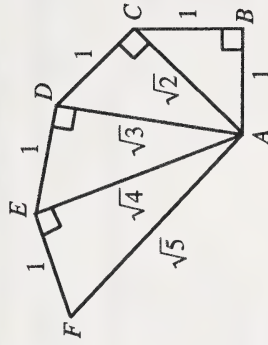
- It is irrational because it cannot be written in the form $\frac{a}{b}$.
 - It is rational because it can be written in the form $\frac{a}{b}$.
 - It is irrational because it is a non-repeating infinite decimal.
 - It is rational because it is an infinite repeating decimal.
 - It is irrational because it cannot be written in the form $\frac{a}{b}$.
 - It is irrational because it cannot be written in the form $\frac{a}{b}$.

2. Since $\sqrt{\frac{4}{9}} = \frac{2}{3}$, it can be written in the form $\frac{a}{b}$.

3. Irrational numbers are numbers that do not repeat or terminate.

Activity 2

Find approximations for square roots of non-perfect squares.



To find $\sqrt{4}$, construct a line of unit length perpendicular to the hypotenuse $\overline{AD}$. Measure $\overline{AE}$.

The length of $\overline{AE}$ is approximately 2 units.

By calculation, the measure of the new hypotenuse is as follows:

$$\begin{aligned}\sqrt{1^2 + (\sqrt{3})^2} &= \sqrt{1 + 3} \\ &= \sqrt{4} \\ &= 2\end{aligned}$$

This value should closely match your measured value.

To find $\sqrt{5}$, construct a line of unit length perpendicular to the hypotenuse $\overline{AE}$. Measure $\overline{AF}$.

The length of $\overline{AF}$ is approximately 2.2 units.

By calculation, the measure of the new hypotenuse is as follows:

$$\begin{aligned}\sqrt{1^2 + (\sqrt{4})^2} &= \sqrt{1+4} \\ &= \sqrt{5} \\ &\doteq 2.2\end{aligned}$$

This value should closely match your measured value.

Activity 3

Evaluate square roots and write the square roots of negative integers as imaginary numbers.

1. $\sqrt{25} = \underline{5}$ because $\underline{5}^2 = \underline{5} \times \underline{5} = 25$

2. $\sqrt{16} = \underline{4}$ because $\underline{4}^2 = \underline{4} \times \underline{4} = \underline{16}$

3. $\sqrt{\frac{9}{25}} = \underline{\frac{3}{5}}$ because $\left(\underline{\frac{3}{5}}\right)^2 = \underline{\frac{3}{5}} \times \underline{\frac{3}{5}} = \underline{\frac{9}{25}}$

4. a. 5.20

b. 7.75

c. -12.73

d. 1.53

e. $-\frac{4}{7}$

5. a. $\sqrt{-64} = \sqrt{64} \times \sqrt{-1}$
 $= \sqrt{64} \times \sqrt{i^2}$
 $= 8 \times i$
 $= 8i$

b. $\sqrt{-75} = \sqrt{75} \times \sqrt{-1}$
 $= \sqrt{75} \times \sqrt{i^2}$
 $\doteq 8.66 \times i$
 $\doteq 8.66i$

c. $\sqrt{-81} = \sqrt{81} \times \sqrt{-1}$
 $= \sqrt{81} \times \sqrt{i^2}$
 $= 9 \times i$
 $= 9i$

d. $\sqrt{-2000} = \sqrt{2000} \times \sqrt{-1}$
 $= \sqrt{2000} \times \sqrt{i^2}$
 $\doteq 44.72 \times i$
 $\doteq 44.72i$

6. $\sqrt{225} = 15$

Each side of the square measure 15 cm.

7. The radical sign $\sqrt{\quad}$ denotes the principal or positive square root. Therefore, $\sqrt{25} = 5$ which is positive.

8. The side of a square is a measure. It represents length. Length must have a positive value.

Activity 4

Recognize π as a special irrational number.

1. In 470 A.D, π was calculated to be the following:

$$\frac{355}{113} = 3.141\,592\,92$$

In 530 A.D, π was calculated to be the following:

$$\frac{3927}{1250} = 3.1416$$

In 1220 A.D, π was calculated to be the following:

$$\frac{864}{275} = 3.141\,818\,182$$

The present value of π to ten digits is 3.141 592 654.

No, the values did not improve over time. The Chinese had the closest value in 470 A.D.

$$\begin{aligned} 2. \quad c &= \pi d \\ &= 3.141\,592\,654 \times 12 \\ &= 37.699\,111\,84 \\ &\doteq 37.70 \text{ cm} \end{aligned}$$

The circumference of the circle is about 37.70 cm.

Extra Help

- | | |
|------------------|---------------|
| 1. a. irrational | b. irrational |
| c. rational | d. rational |
| e. rational | |
-
- | | |
|---|--|
| 2. a. $\sqrt{-7} = \sqrt{7}i$ | b. $-\sqrt{-8} = -\sqrt{8}i$ |
| c. $-\sqrt{27} = -\sqrt{3 \times 3 \times 3}$ | d. $\sqrt{125} = \sqrt{5 \times 5 \times 5}$ |
| $= -\sqrt{9} \times \sqrt{3}$ | $= \sqrt{25} \times \sqrt{5}$ |
| $= -3\sqrt{3}$ | $= 5\sqrt{5}$ |

Extensions

$$\begin{aligned} 1. \quad c^2 &= a^2 + b^2 \\ c^2 &= 100^2 + 108^2 \\ c^2 &= 10\,000 + 11\,664 \\ c^2 &= 21\,664 \\ \sqrt{c^2} &= \sqrt{21\,664} \\ c &\doteq 147.186\,9559 \\ &\doteq 147 \end{aligned}$$

The distance is about 147 m from the tee box to the flag.

$$2. \quad c^2 = a^2 + b^2$$

$$3^2 = a^2 + 1.5^2$$

$$9 = a^2 + 2.25$$

$$a^2 = 6.75$$

$$\sqrt{a^2} = \sqrt{6.75}$$

$$a \doteq 2.598\,076\,211$$

$$\doteq 2.6$$

The ball travels about 2.6 m.

$$3. \quad a. \quad x^2 = 25^2 - 10^2$$

$$x^2 = 625 - 100$$

$$x^2 = 525$$

$$\sqrt{x^2} = \sqrt{525}$$

$$x \doteq 22.912\,878\,47$$

$$\doteq 22.9$$

Side x is about 22.9 m in length.

$$b. \quad m^2 = (\sqrt{7})^2 + 3^2$$

$$m^2 = 7 + 9$$

$$m^2 = 16$$

$$\sqrt{m^2} = \sqrt{16}$$

$$m = 4$$

Side m is 4 units in length.

c. Since the area is 240 cm^2 , one side measures $\sqrt{240}\text{ cm}$.

$$c^2 = 25^2 - (\sqrt{240})^2$$

$$c^2 = 625 - 240$$

$$c^2 = 385$$

$$\sqrt{c^2} = \sqrt{385}$$

$$c \doteq 19.621\,416\,87$$

$$\doteq 19.6$$

Side c is about 19.6 cm in length.



Exploring Topic 3

4. Let $\sqrt{3} = \frac{a}{b}$, where $a, b \in I$ and $b \neq 0$.

The expression $\frac{a}{b}$ is a rational number in lowest terms. Square both sides.

$$(\sqrt{3})^2 = \left(\frac{a}{b}\right)^2$$

$$3 = \frac{a^2}{b^2}$$

By cross multiplication, $3b^2 = a^2$.

The number 3 is a factor of $3b^2$.

Since a is a multiple of 3, a can be written as $3n$, $n \in I$.

$$\text{Therefore, } a^2 = (3n)^2$$

$$9n^2 = 3b^2$$

$$a^2 = 9n^2$$

$$3n^2 = b^2$$

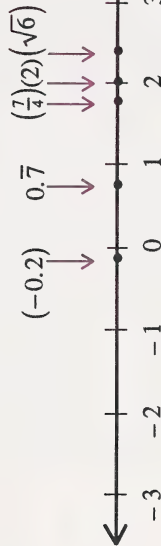
Since the left number has 3 as a factor, b is a multiple of 3. So a and b have a common factor. This contradicts the statement that $\frac{a}{b}$ is in lowest terms. Therefore, $\sqrt{3}$ is irrational.

Activity 1

Identify real numbers as the union of rational and irrational numbers.

1. a. rational b. irrational
- c. rational d. irrational
- e. irrational f. rational
- g. rational h. irrational

2.



Activity 2

Extensions

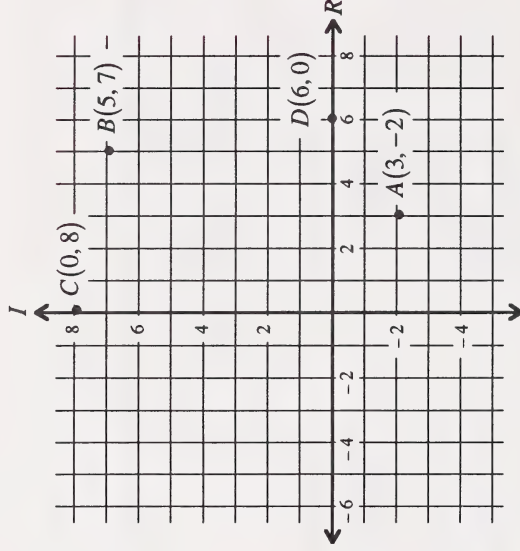
Identify complex numbers as the union of real and imaginary numbers.

The complex number system is the union of the real numbers and the imaginary numbers.

Extra Help

1. No
2. Yes
3. No
4. Yes
5. No
6. Yes
7. Yes
8. Yes

1. a. $A(3, -2)$ b. $B(5, 7)$
c. $C(0, 8)$ d. $D(6, 0)$



2. a. $3+7i$ b. $5i$
c. 9 d. $-2-7i$

NOTES

NOTES



Mathematics 10

Student Module
Unit 1

L.R.D.C.
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